

Leyes de Kirchoff

- Aplicables sin fallos apreciables solo si:

$$L_{max} = \frac{\lambda}{100} = \frac{c}{100g}$$

La Transformada de Laplace

$$V(s) = \mathcal{L}\{V(t)\} = \int_0^{\infty} V(t) e^{-st} dt$$

Transformadas comunes

$$k \mu(t) \xrightarrow{\mathcal{L}} \frac{k}{s}$$

$$\delta(t) \xrightarrow{\mathcal{L}} 1$$

$$k e^{-at} \xrightarrow{\mathcal{L}} \frac{k}{s+a}$$

$$k e^{+at} \xrightarrow{\mathcal{L}} \frac{k}{s-a}$$

$$A \cos(\omega_0 t) \xrightarrow{\mathcal{L}} \frac{As}{s^2 + \omega_0^2}$$

$$A \sin(\omega_0 t) \xrightarrow{\mathcal{L}} \frac{A \omega_0}{s^2 + \omega_0^2}$$

$$\frac{d}{dt} f(t) \xrightarrow{\mathcal{L}^{-1}} sF(s) - f(t)|_{t=0}$$

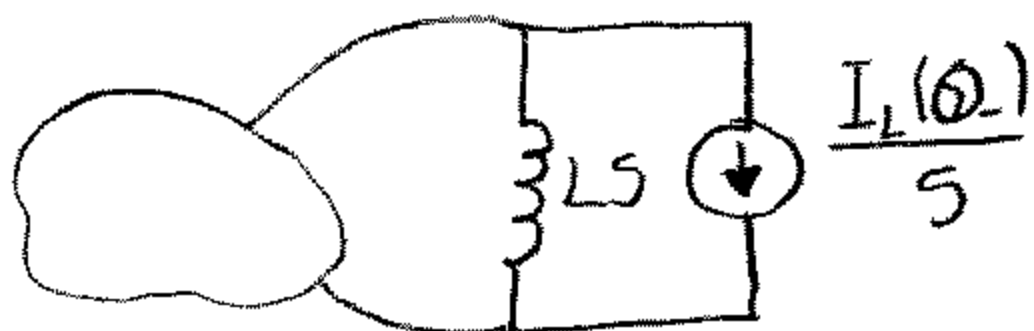
$$\frac{d^2 f}{dt^2} \xrightarrow{\mathcal{L}^{-1}} s^2 F(s) - s f(0) - \frac{df}{dt} \Big|_{t=0}$$

$$\int_0^t f(\tau) d\tau \xrightarrow{\mathcal{L}^{-1}} \frac{F(s)}{s}$$

Polos dobles: Multiplican por (t)

$$t \mu(t) \xrightarrow{\mathcal{L}^{-1}} \frac{1}{s^2}$$

Generadores de C.I



Estabilidad del circuito

1) $D(s) = as + b \rightarrow$ Completo y mismo signo

2) $D(s) = as^2 + bs + c \rightarrow$ Completo y mismo signo

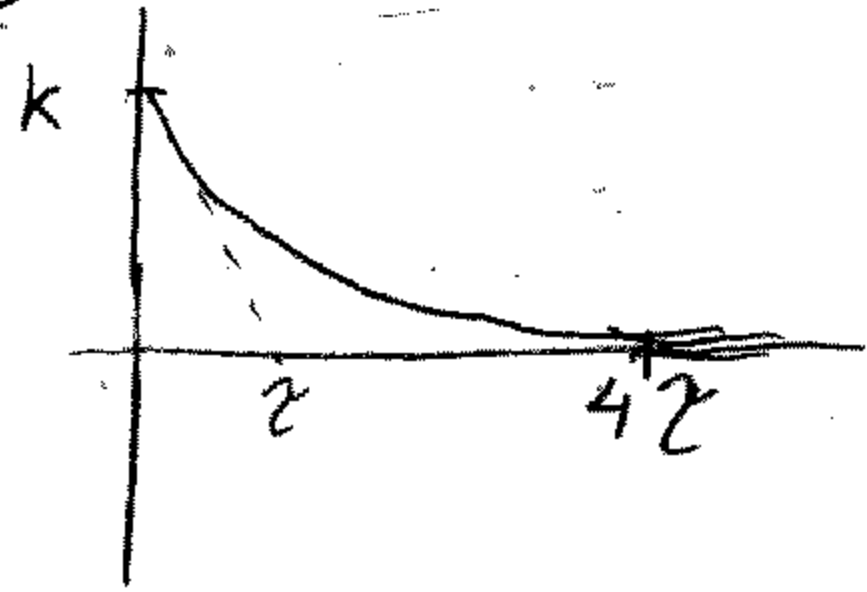
3) $D(s) = as^3 + bs^2 + cs + d \rightarrow$ Completo, mismo signo y $bc > ad$

Nota: Si $bc = ad \Rightarrow$ OSCILADOR

Duración del régimen transitorio

$$f(t) = Ke^{-t/\tau} u(t)$$

$$- \boxed{4\tau}$$



- Caso polos dobles: $\boxed{\tau = 5\tau}$

Tipos de circuitos

- Con respuesta nula $\Rightarrow H(s) = cte \Rightarrow$ cancelación polo-cero

- De 1^{er} orden (solo un C o L)

$$C: \boxed{\tau = R_{TH} \cdot C}$$

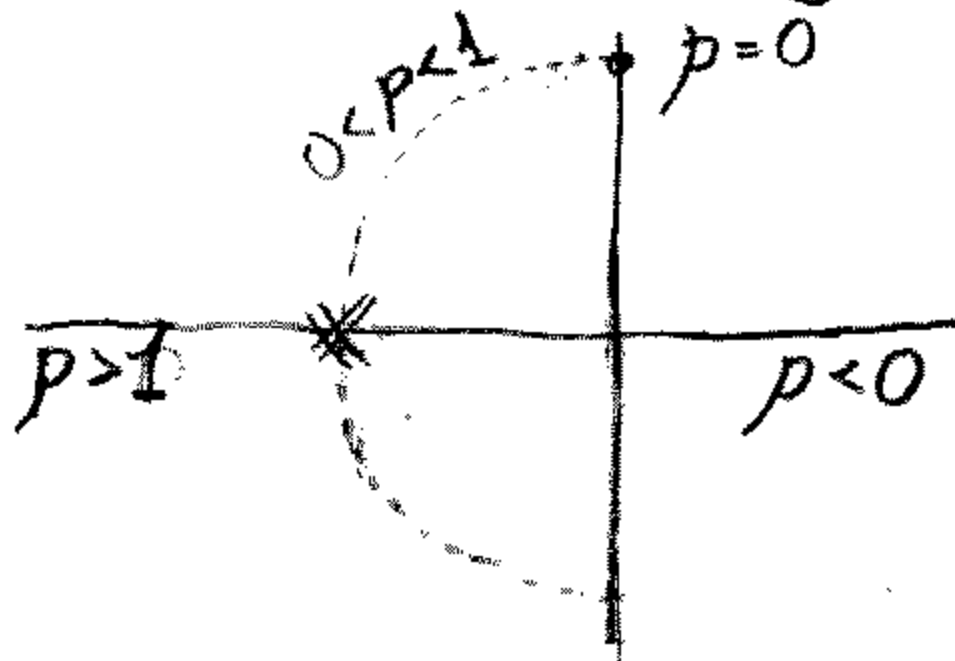
$$L: \boxed{\tau = \frac{L}{R_{TH}}}$$

- De 2^o orden (2C y/o 2L)

$$\boxed{H(s) = \frac{N(s)}{a s^2 + b s + c} = \frac{N(s)}{s^2 + 2p\omega_0 s + \omega_0^2}}$$

p : coef. amortiguam.
 ω_0 : puls. natural

- Respuesta que generan



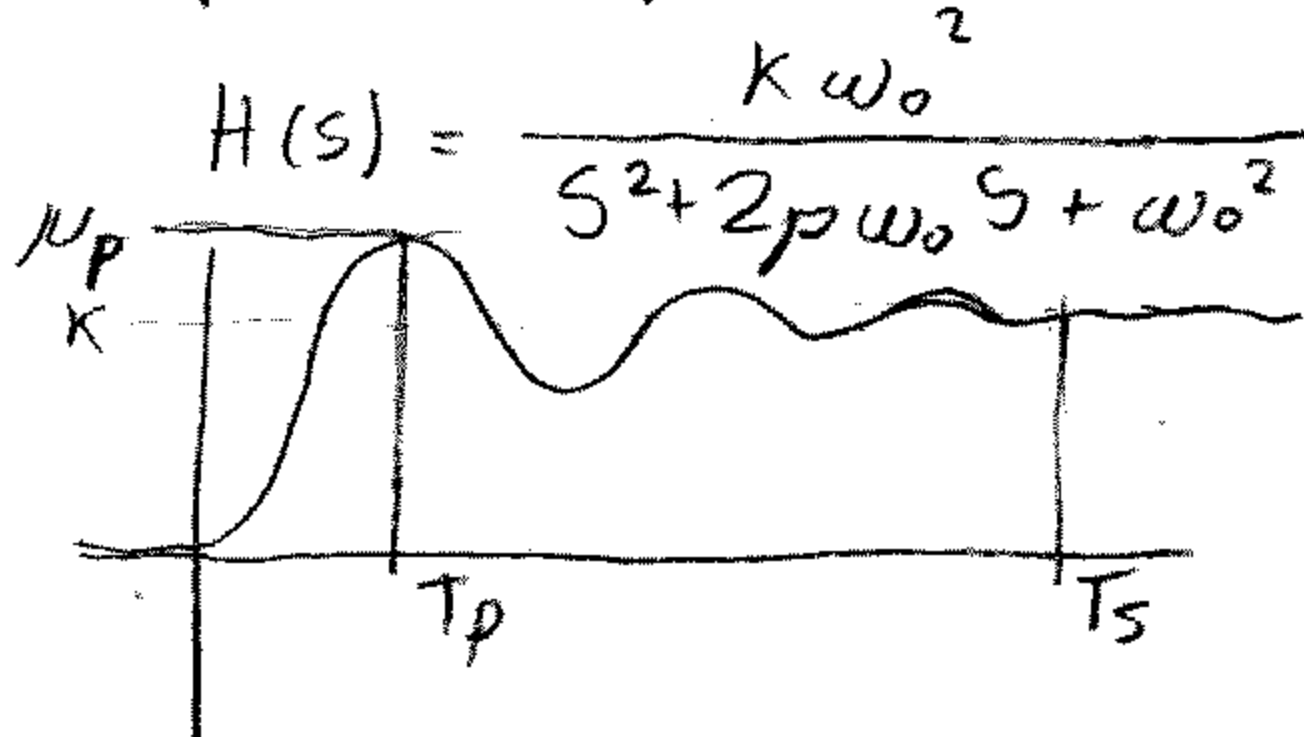
Resp. Propia:

$$0 < p < 1 \Rightarrow Ke^{-p\omega_0 t} \cos(\omega_0 \sqrt{1-p^2} t + \alpha)$$

Duración Transitorio

$$\tau = \frac{1}{p\omega_0} \quad 4\tau$$

- Respuesta Propia obtenida a menudo



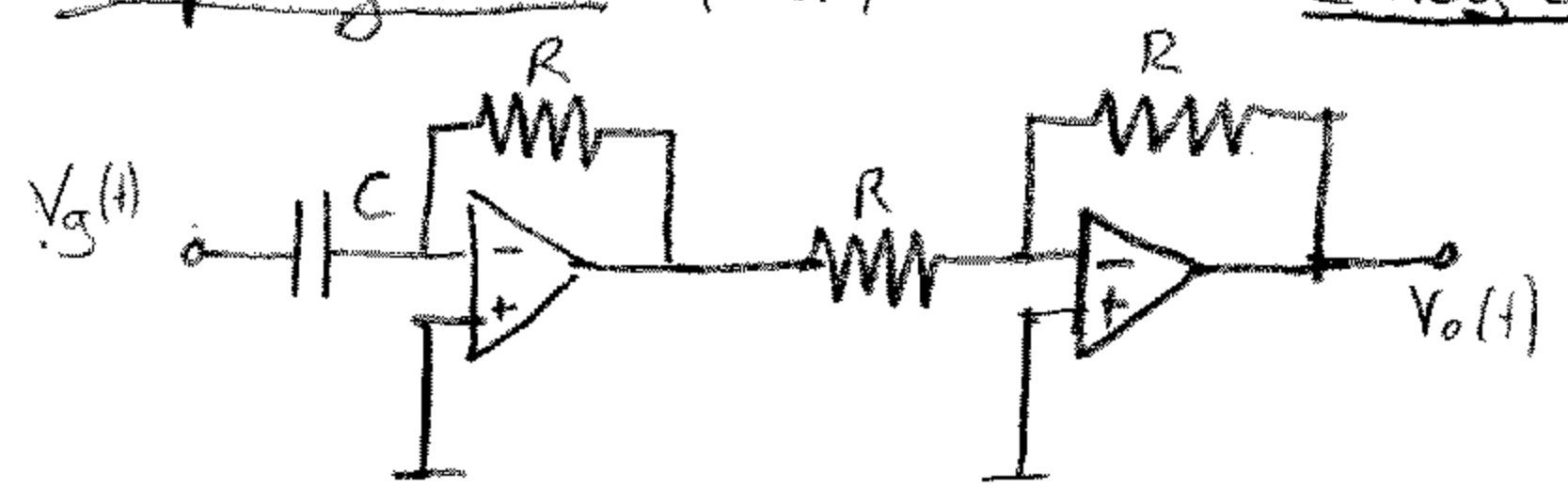
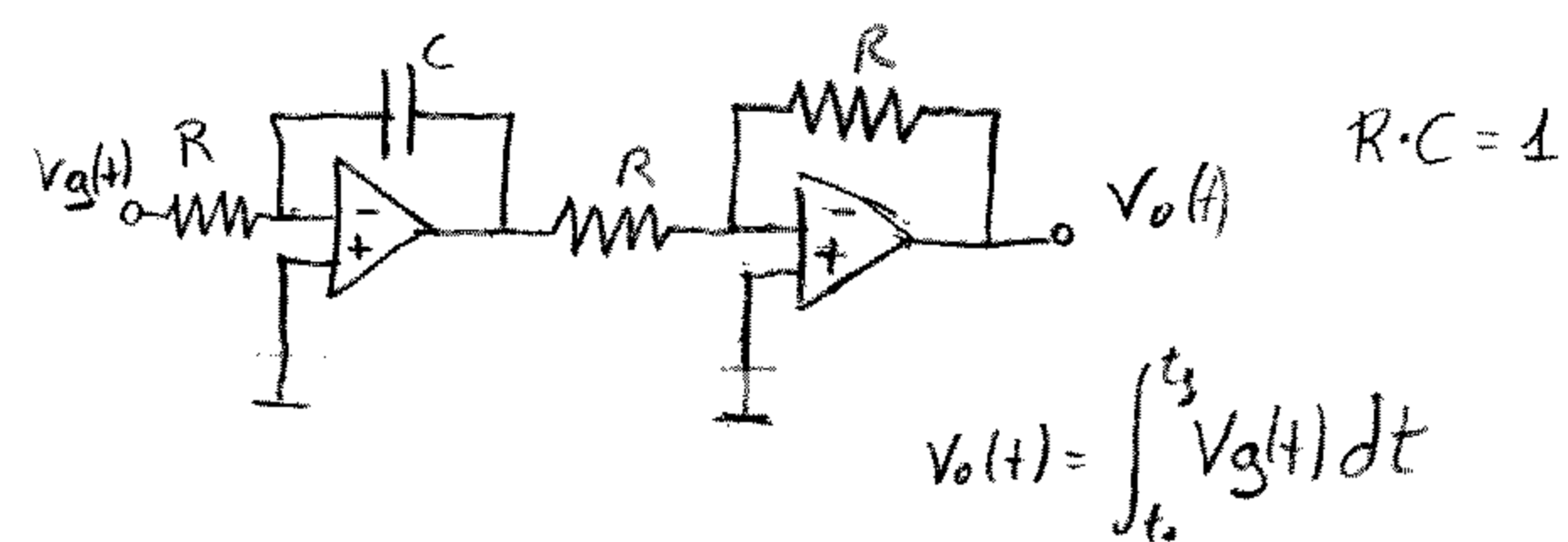
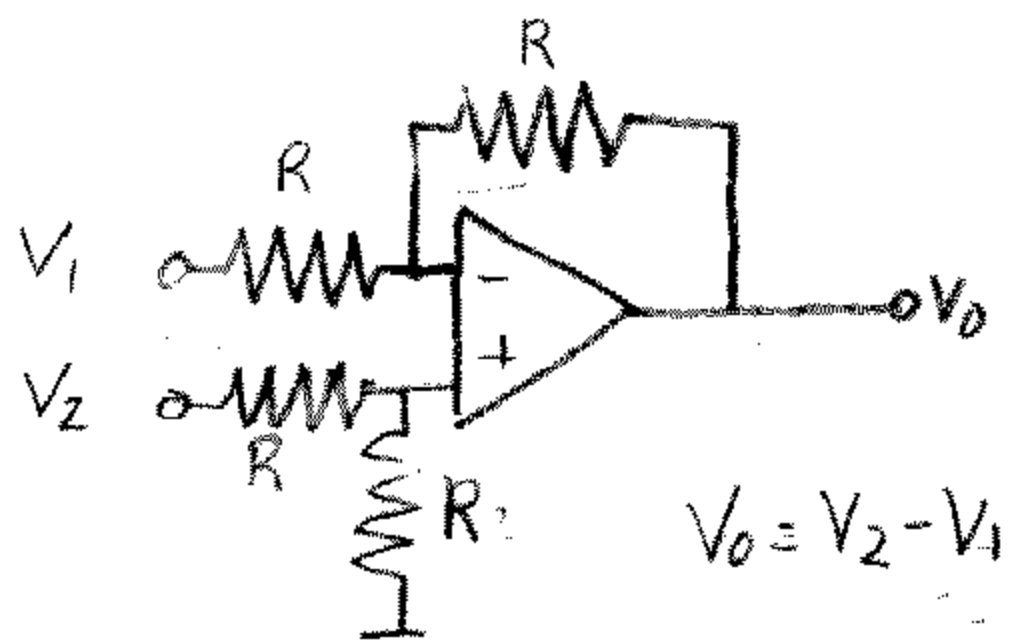
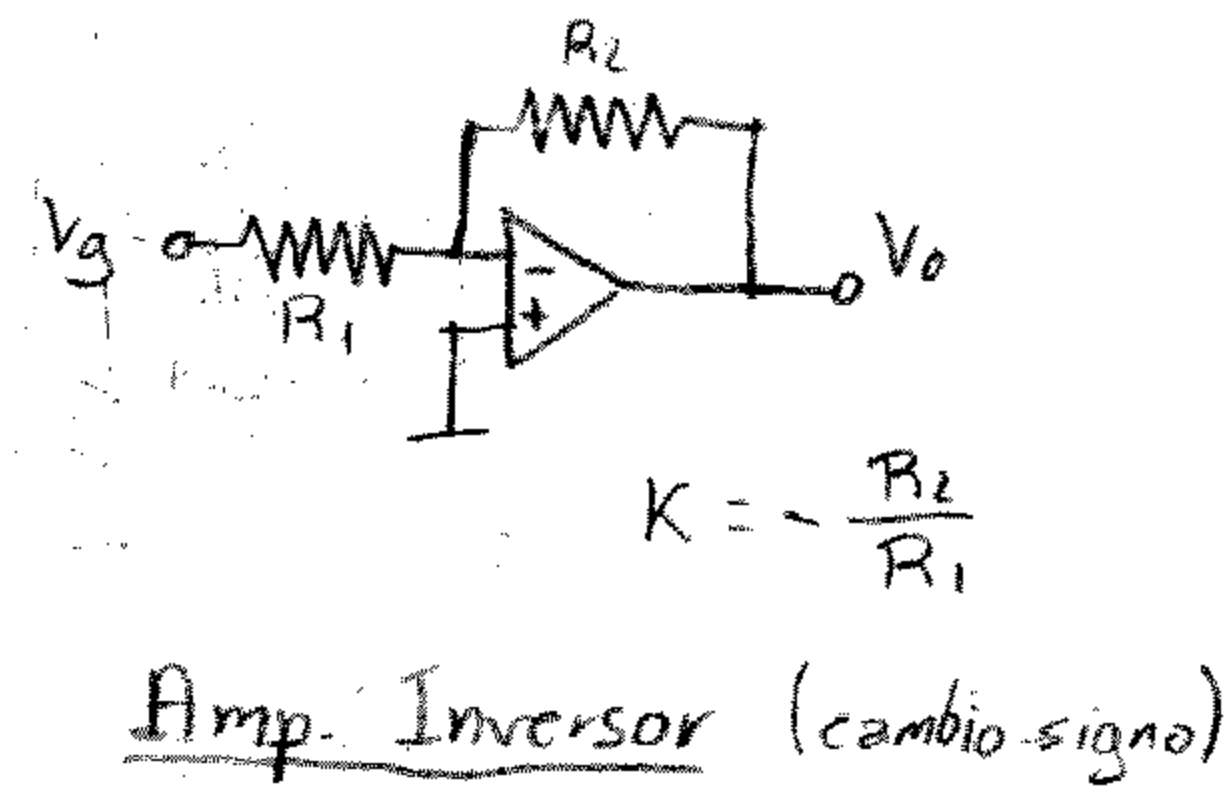
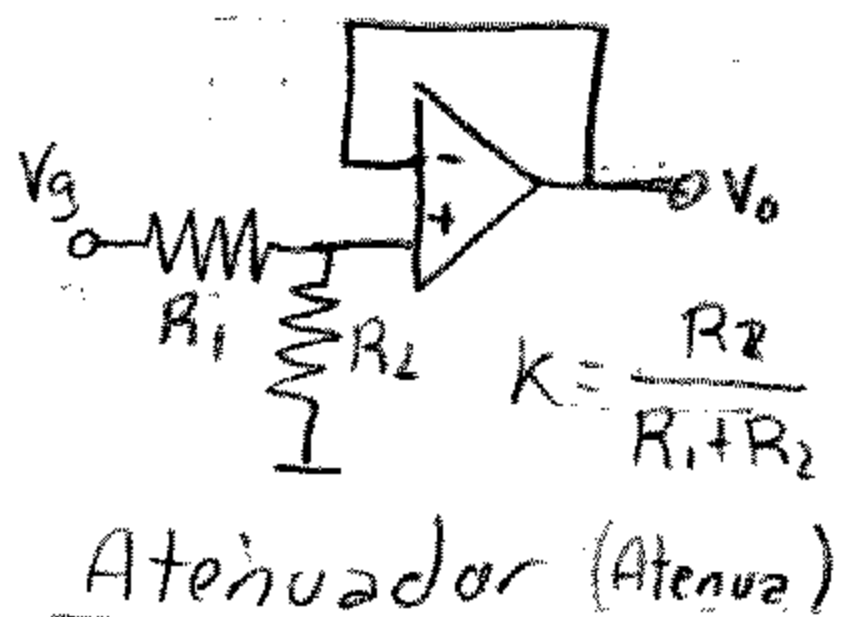
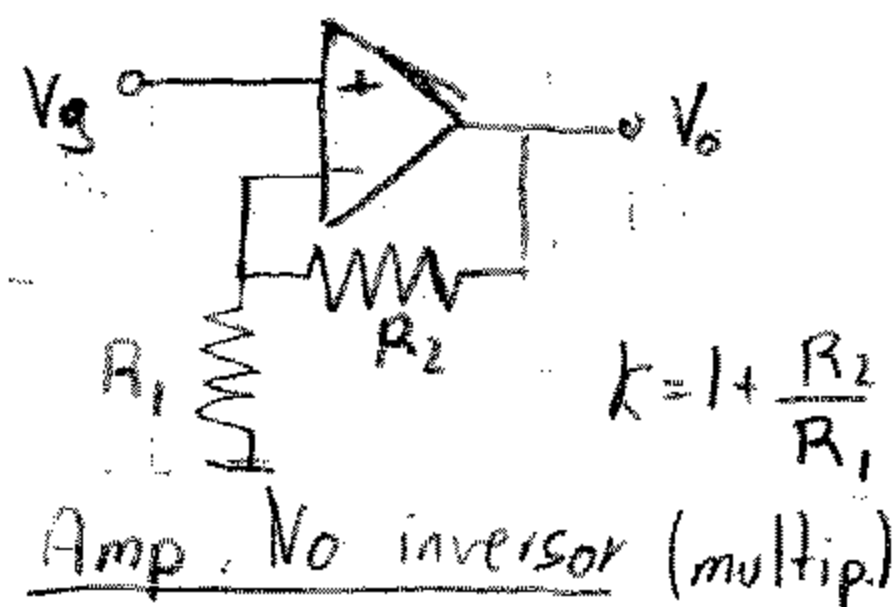
$$H(s) = \frac{K\omega_0^2}{s^2 + 2p\omega_0 s + \omega_0^2}$$

$$T_p = \frac{\pi}{\omega_0 \sqrt{1-p^2}}$$

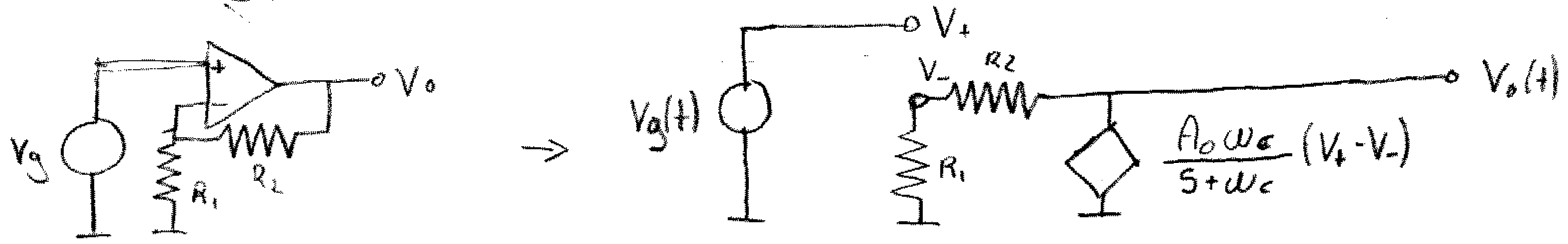
$$T_s = \frac{4}{p\omega_0}$$

$$\mu_p = (1 + e^{-\pi p / \sqrt{1-p^2}}) K$$

AO's



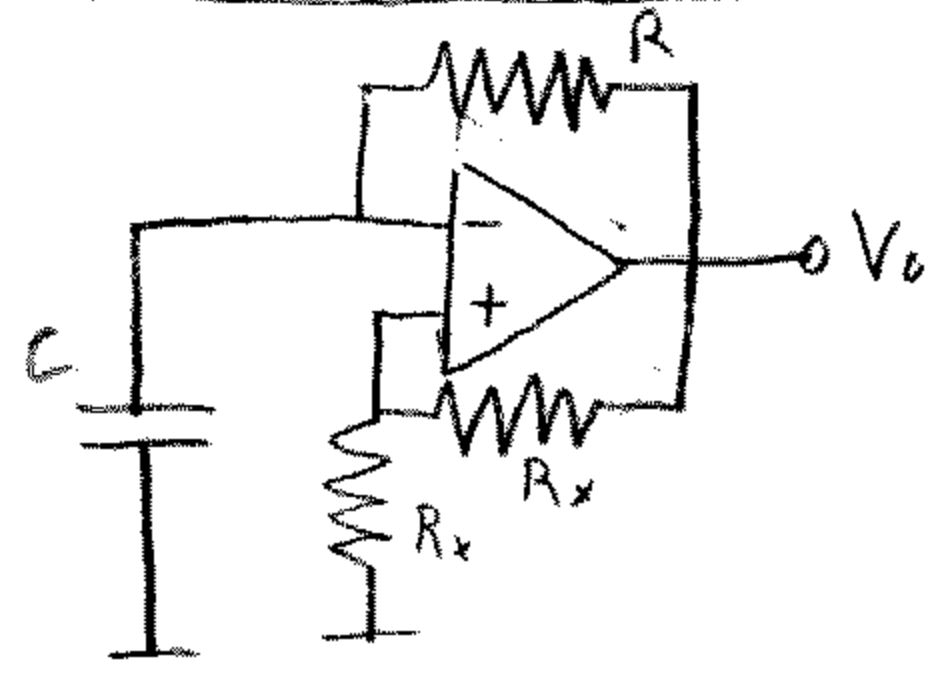
Modelo ampliado del AO



$\omega_c = 2\pi f_c$; $GB = A_0 f_c$;

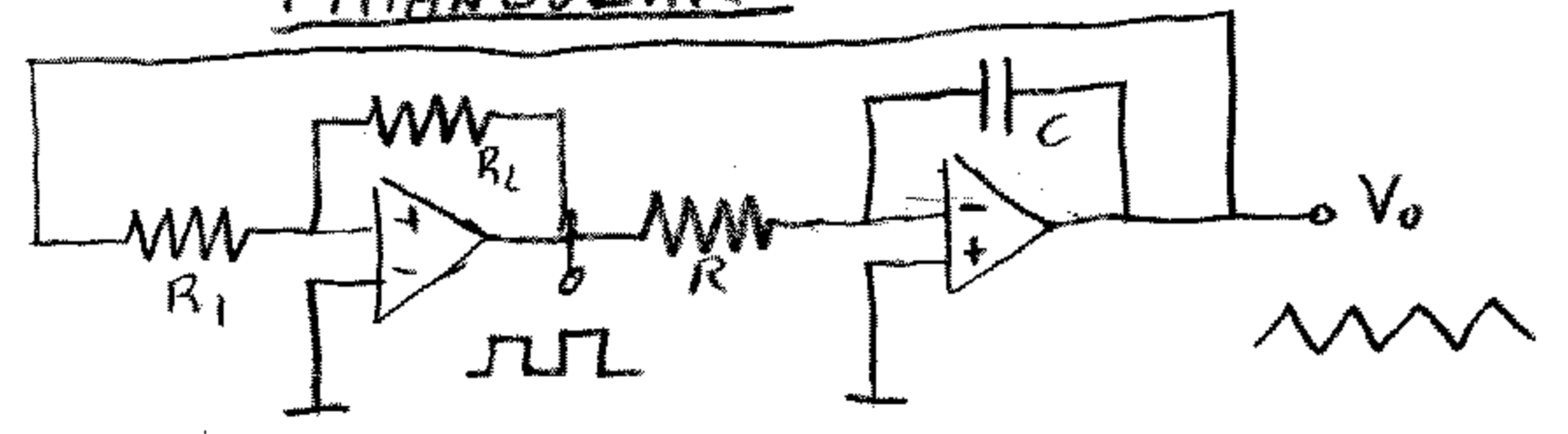
$Z = \frac{1 + R_2/R_1}{A_0 \omega_c}$ (En este caso)

GENERADOR SEÑAL CUADRADA



$R_x \text{ tipico} = 10k$
 $f_{osc} = \frac{0.45}{RC}$

TRIANGULAR *



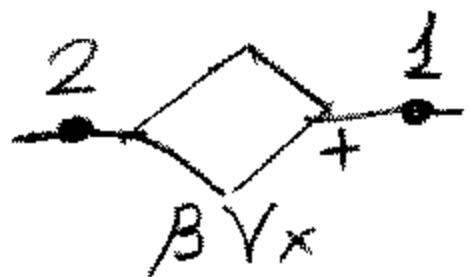
Amplitud = $\pm \frac{R_1}{R_2} V_{cc}$
 $f_{osc} = \frac{1}{4RC} \cdot \frac{R_2}{R_1}$

* Funciona gracias al ciclo de histéresis (son inestables)

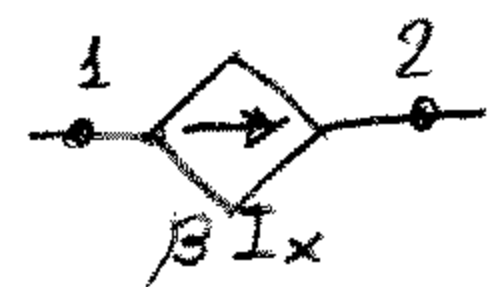
PSPICE



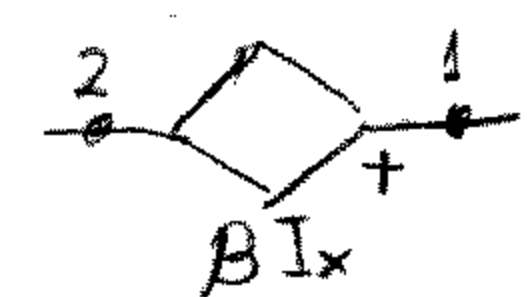
g1 2 1 3 4 B



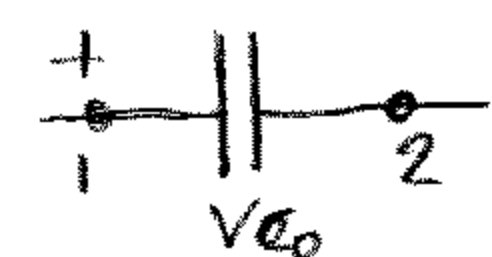
e1 1 2 3 4 B



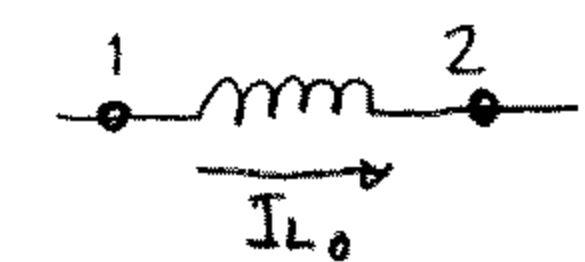
g1 1 2 vs B
vs 3 4 dc 0



e1 1 2 vs B
vs 3 4 dc 0



C1 1 2 100u ic = 5



L1 1 2 1m ic = 1e-2

E/S

.tj v(3) vq

Definir tiempos de evaluación

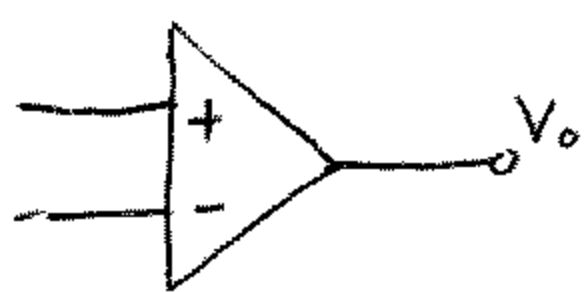
.tran :01 20 vic
.tran .01 20 0 .01 vic

Gráfico

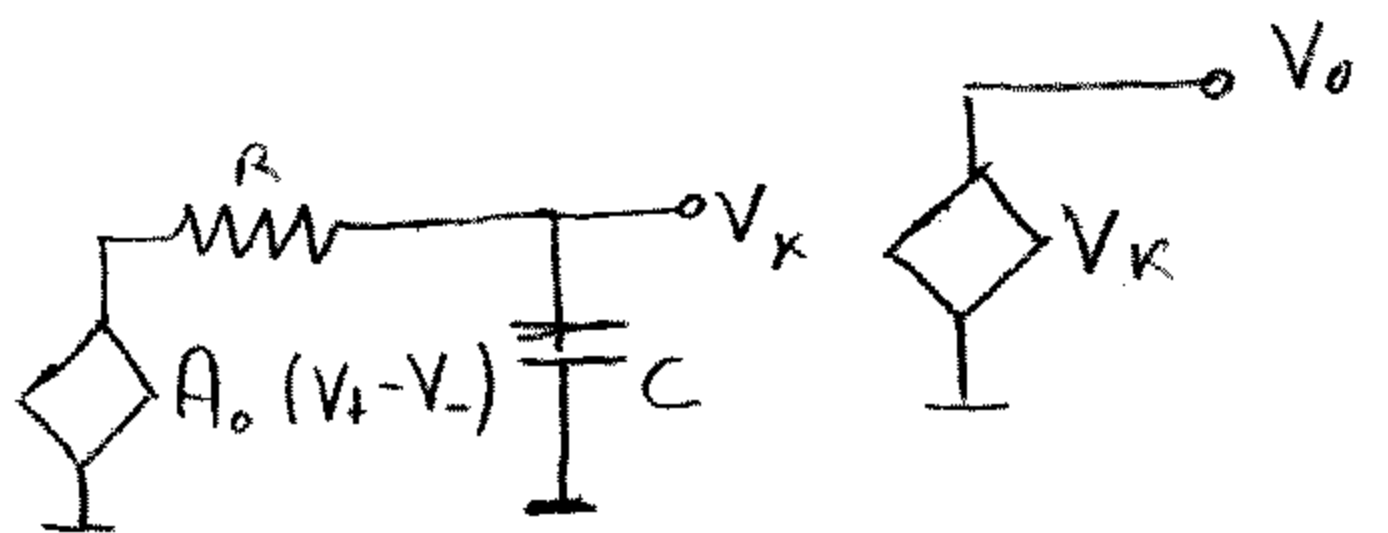
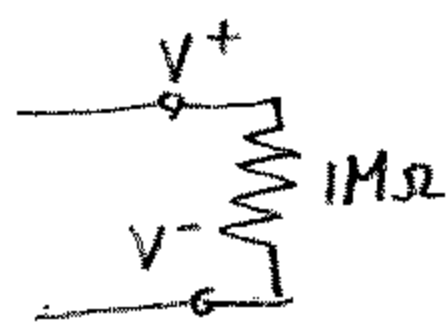
.probe v(3)

Tabla valores

.print tran v(3)



$$\omega = \frac{1}{RC}$$



R.P.S

Vxx 1 0 AC Vm θ°
.AC LIN n f0 fs
.PRINT AC V(1) σ .PROBE V(1)

FOURIER

```
Vg 1 0 PVL( ... )
R1 1 0 1000
.TRAN <<t z
.FOUR f0 V(1)
.END
```

CIRCUITOS EN R.P.S

$V_g = A \cos(\omega_0 t + \theta) \Rightarrow V_o(t)_{R.P.S} = A \cdot |H(\omega_0 j)| \cos(\omega_0 t + \theta + \arg H(\omega_0 j))$

CIRCUITO TRANSFORMADO FASORIAL

$L \rightarrow L\omega_0 j \quad C \rightarrow \frac{1}{C\omega_0 j} \quad V \cos(\omega_0 t + \theta) \rightarrow V e^{j\theta} = V \angle \theta$

* Excitaciones a diferentes frecuencias \Rightarrow Análisis por superposición

POTENCIAS EN CIRCUITOS EN RPS

$V(t)_{RMS} = \sqrt{\overline{v^2(t)}} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v^2(t) dt}$

$P_m = \frac{1}{2} |\bar{V}| \cdot |\bar{I}| \cos(\arg \bar{V} - \arg \bar{I})$

Resistor $\Rightarrow P_m = \frac{1}{2} |\bar{I}_R|^2 R = |\bar{I}_{efj}|^2 R = \frac{|\bar{V}_{efj}|^2}{R}$

Inductor o Condensador $\Rightarrow P_m = 0$

En un bipolo $\Rightarrow P_m = \frac{1}{2} |\bar{I}_L|^2 \operatorname{Re}[Z] = |\bar{I}_{efj}|^2 \cdot \operatorname{Re}[Z]$

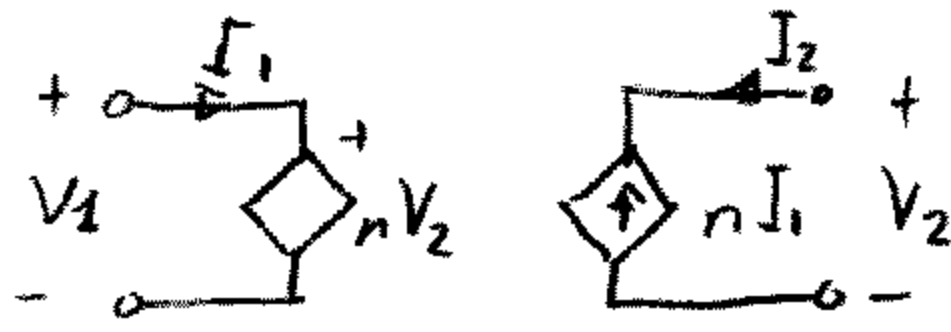
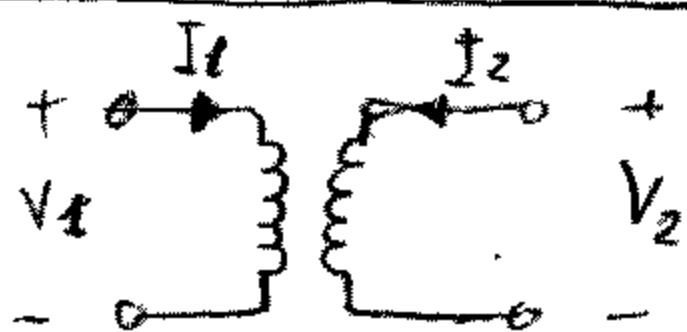
Valor eficaz de una senoide $\Rightarrow V_{efj} = \frac{V_{max}}{\sqrt{2}}$

$\frac{P_m}{P_{ap}} = \cos(\arg \bar{V} - \arg \bar{I})$

$|\bar{I}_{efj}| = \frac{P_{ap}}{V_{efj}}$

$\arg Z = \arg \bar{V} - \arg \bar{I}$

EL TRANSFORMADOR

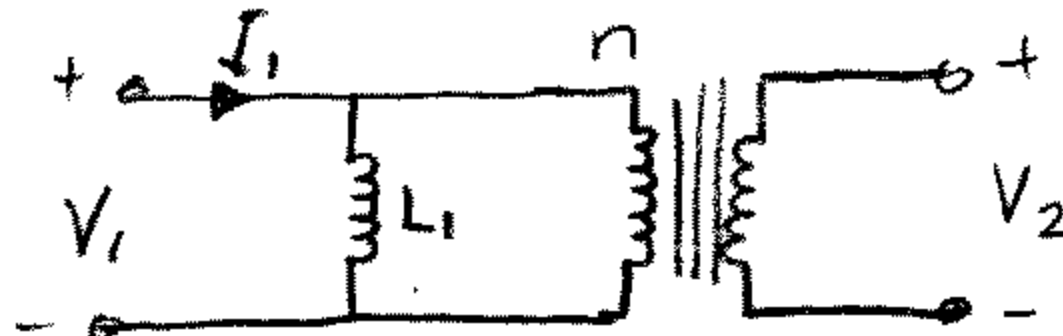
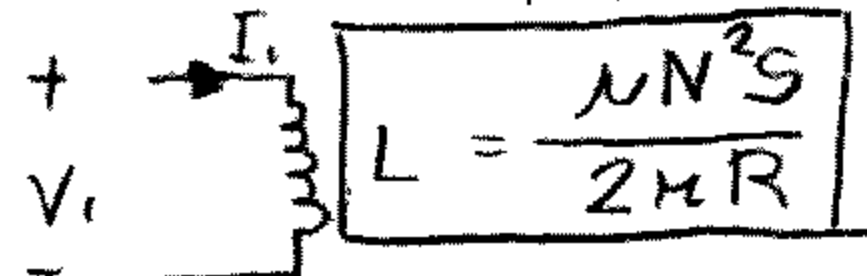


$V_1 = n V_2$
 $n I_1 = -I_2$



\Rightarrow El primario ve un resistor de $n^2 R$

Realización práctica



$n = \frac{N_1}{N_2}$

$V_1 = n V_2$

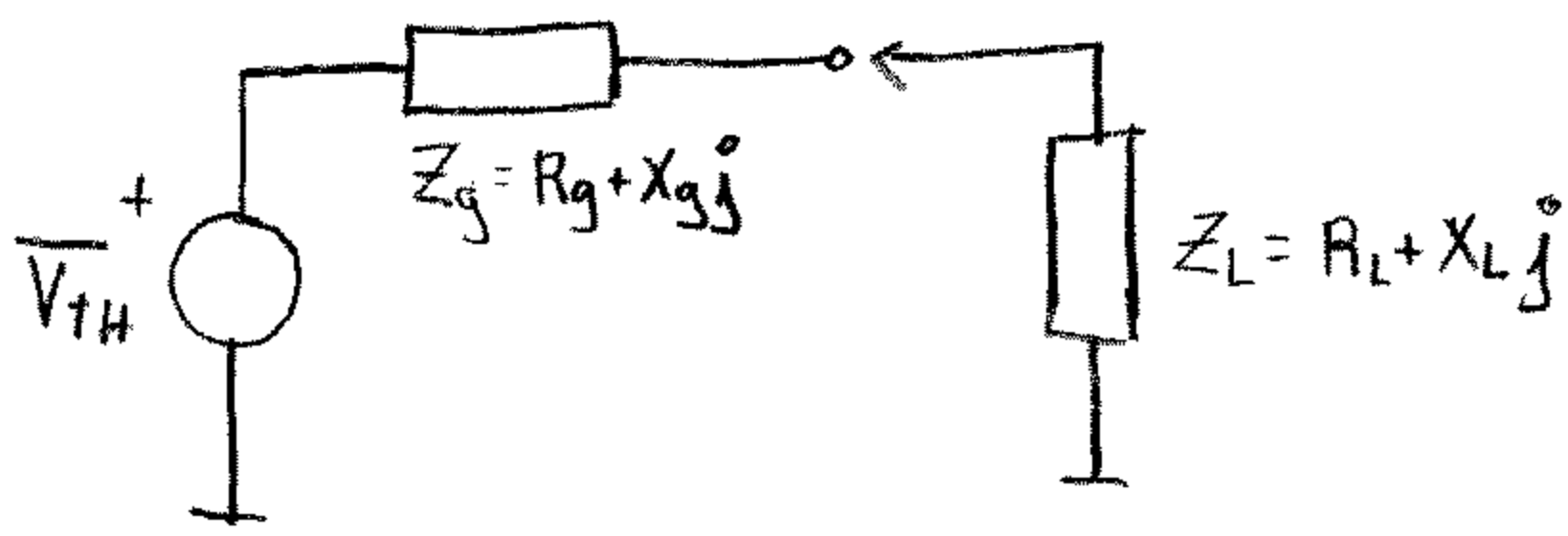
TRANSFORMACIÓN SERIE-PARALELO

$\frac{1}{R_p} = \frac{R_s}{R_s^2 + X_s^2}$
 $\frac{1}{X_p} = \frac{X_s}{R_s^2 + X_s^2}$

PARALELO-SERIE

$R_s = \frac{R_p X_p^2}{R_p^2 + X_p^2}$
 $X_s = \frac{R_p^2 X_p}{R_p^2 + X_p^2}$

MÁXIMA TRANSFERENCIA DE POTENCIA



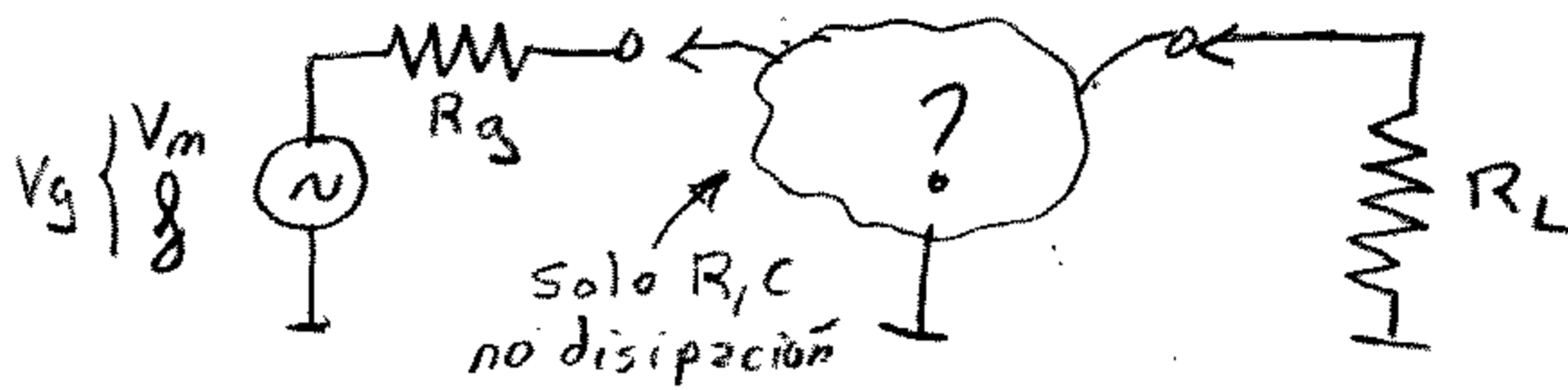
Potencia entregada máxima \Leftrightarrow

$$Z_L = Z_g^* \quad R_L = R_g \quad ; \quad X_L = -X_g$$

$$P_L = \frac{|V_{TH}|^2}{8R_g}$$

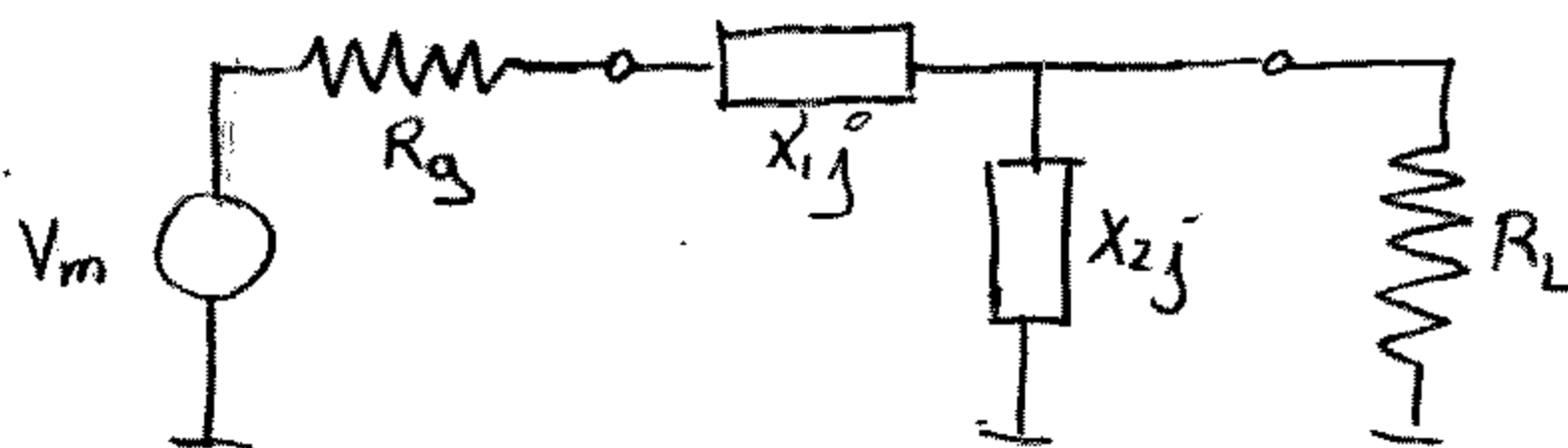
Máx pot. que se puede obtener de un generador de $R_{int} = R_g$

- Redes adaptadoras



$$P_{max} = \frac{V_m^2}{8R_g}$$

Queremos que $P_L = P_{max}$



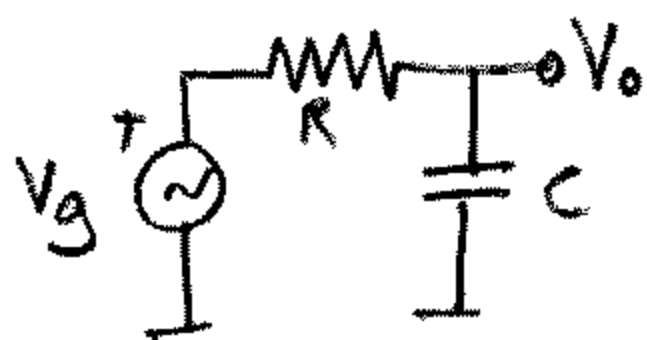
$$X_2 = \pm R_L \sqrt{\frac{R_g}{R_L - R_g}} \quad \Leftrightarrow R_L > R_g$$

$$X_1 = \mp R_g \sqrt{\frac{R_L - R_g}{R_g}}$$

$$R_L < R_g \Leftrightarrow \begin{cases} X_2 = \pm R_g \sqrt{\frac{R_L}{R_g - R_L}} \\ X_1 = \mp R_L \sqrt{\frac{R_g - R_L}{R_L}} = \mp \frac{R_L R_g}{X_2} \end{cases}$$

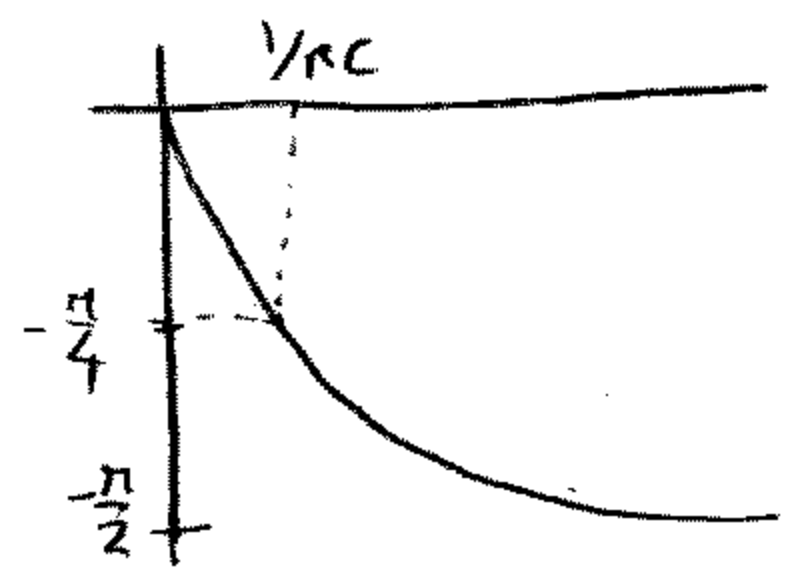
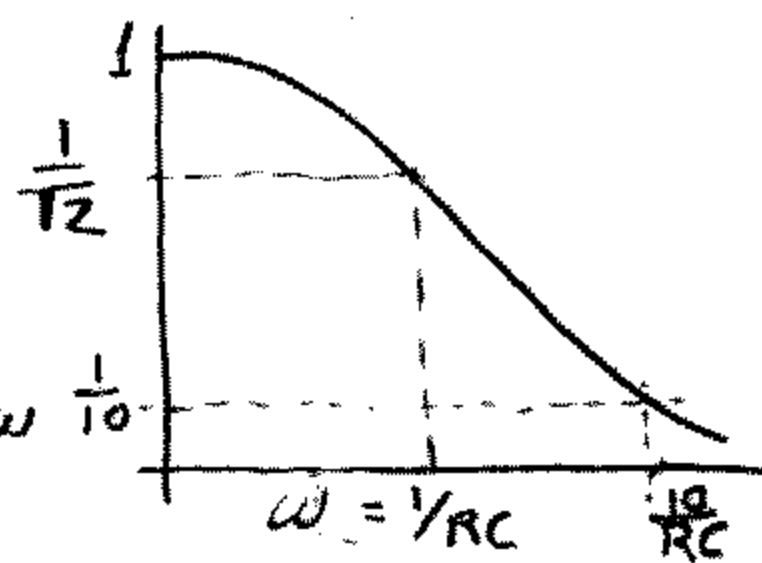
CIRCUITOS SENSIBLES A FRECUENCIA

- Filtro paso-bajo

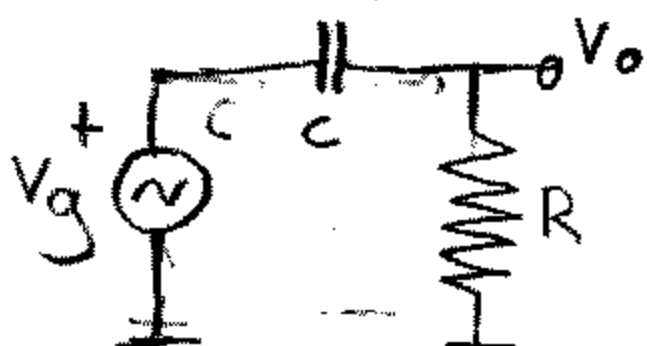


$$\frac{|V_o|}{|V_g|} = \frac{1}{\sqrt{1 + R^2 C^2 \omega^2}}$$

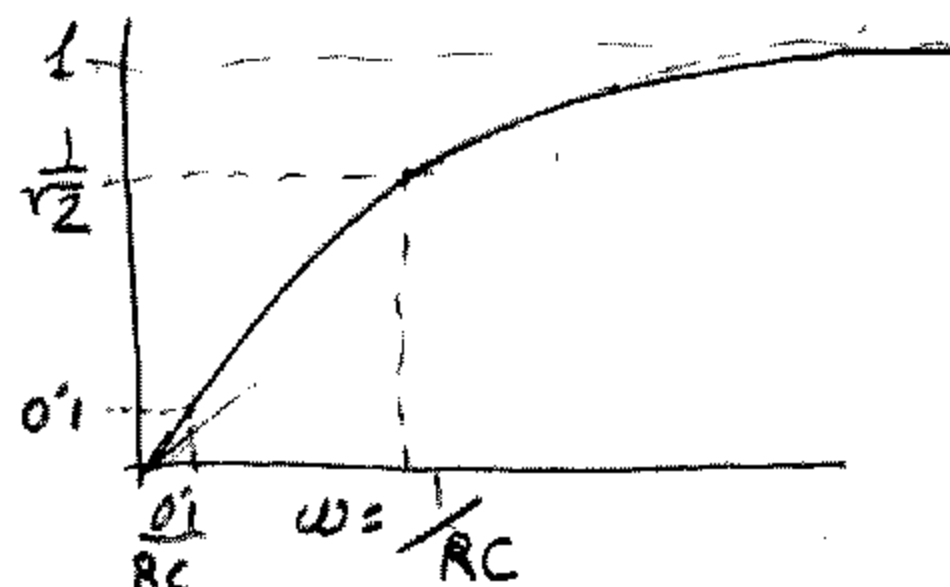
$$\arg V_o = \arg V_g - \arctan RC\omega$$



- Filtro paso-alto

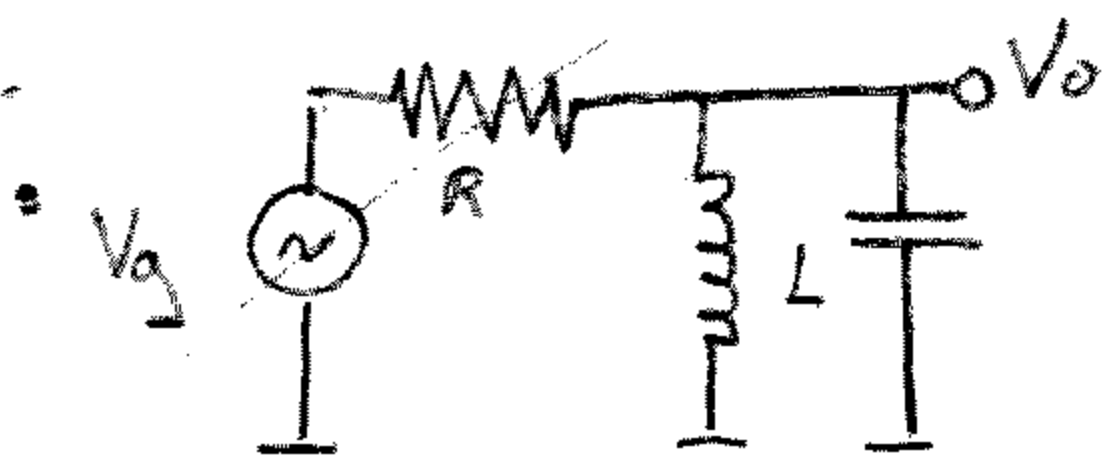


$$\frac{|V_o|}{|V_g|} = \frac{RC\omega}{\sqrt{1 + R^2 C^2 \omega^2}}$$

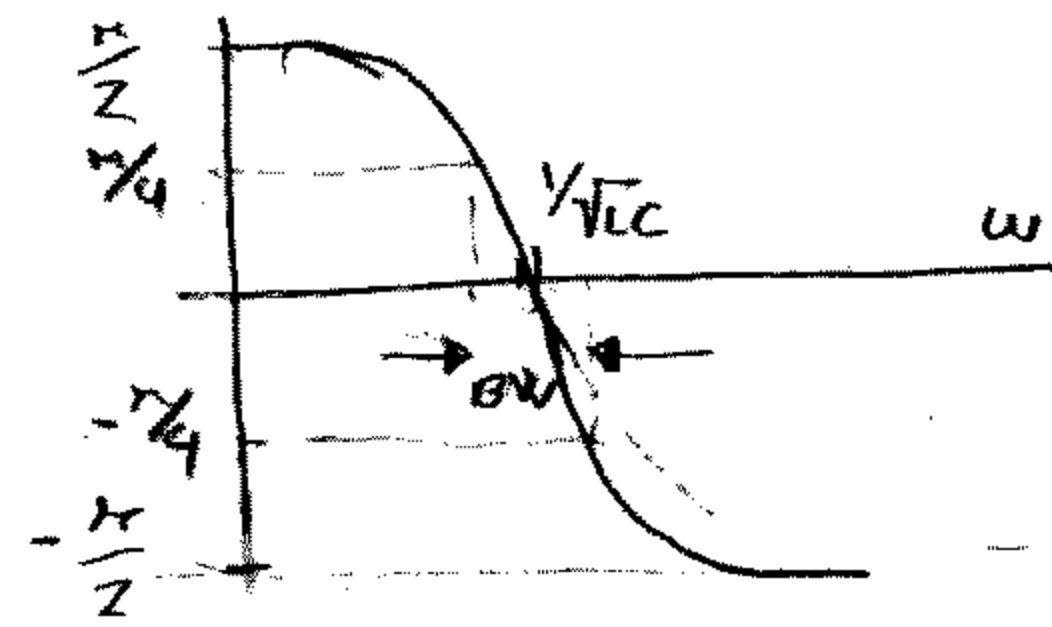
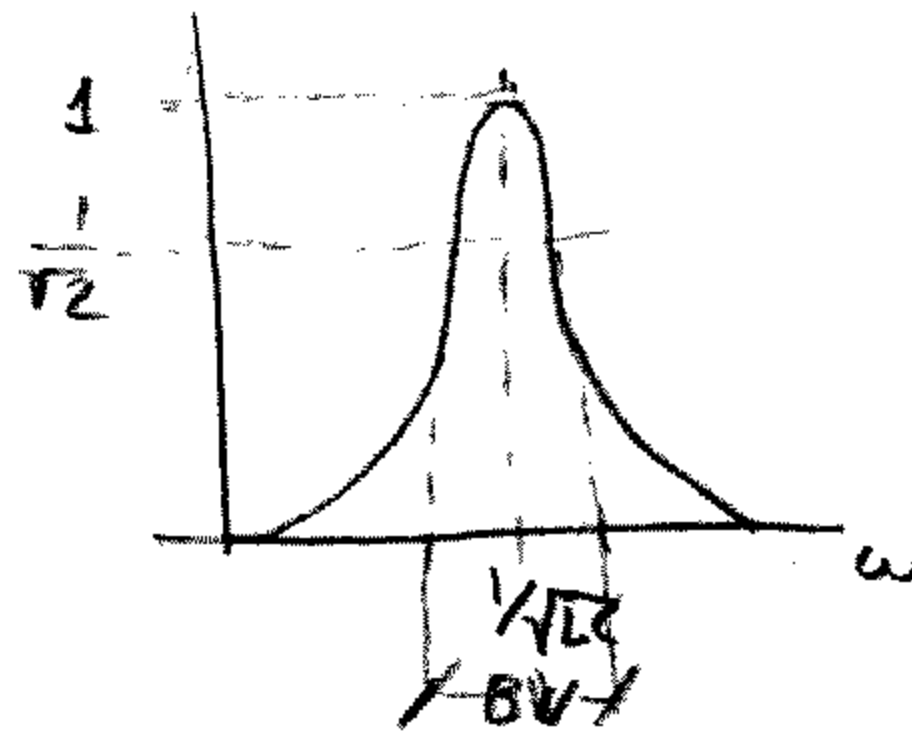


o Doble filtro paso-alto $\Rightarrow f_c = \frac{0.1}{RC}$

- Filtro paso-banda

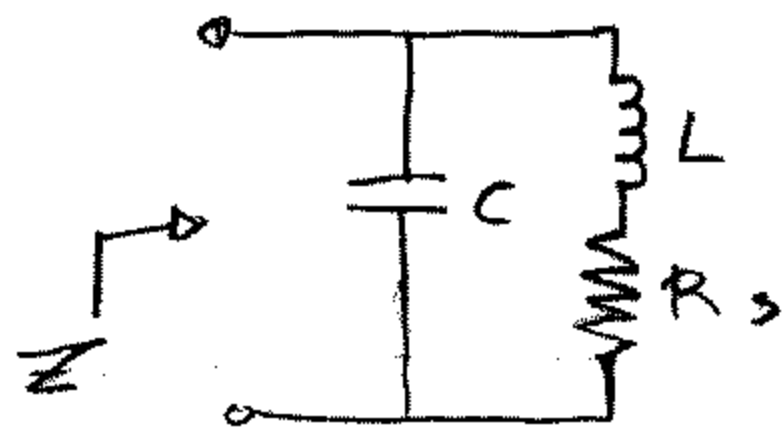


$$BW = \frac{1}{CR} \quad Q = \frac{\omega_r}{BW}$$

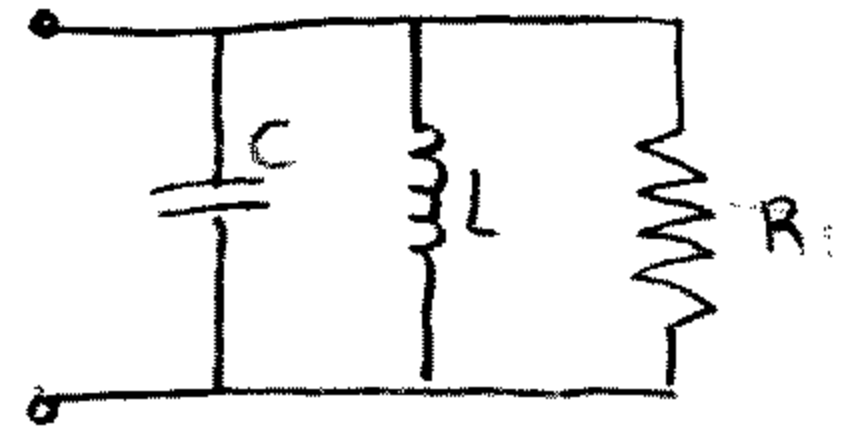


- Bobina real

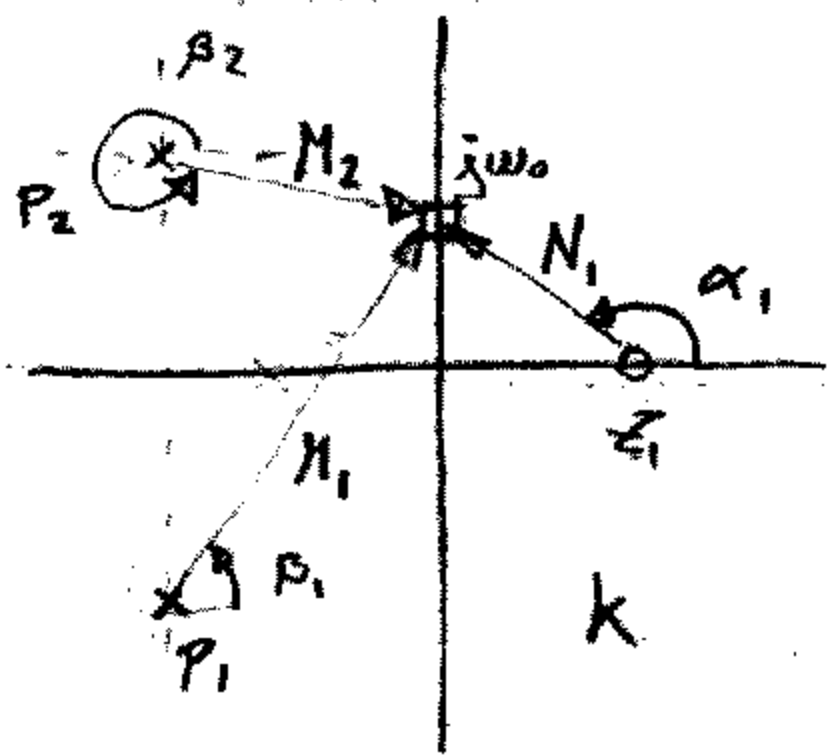
$$\omega_R = \omega_0 \sqrt{1 - \frac{1}{Q^2}} \approx \omega_0 \quad \downarrow Q > 5$$



$$Q_{L\omega_0} = \frac{L\omega}{R_s} \quad R = \frac{L}{R_s C}$$



- Análisis por técnicas gráficas



$$|H(j\omega_0)| = k \frac{N_{\omega_0}}{M_1 \cdot M_2}$$

$$\arg(H(j\omega_0)) = \begin{cases} 0 \\ \pi \end{cases} + \alpha_1 - \beta_1 - \beta_2 + \arg(K)$$

Gráfica polos-ceros

- 1/ Cero en el origen
 - bloquea bajas frec
 - Aporta $\frac{\pi}{2} \forall \omega$



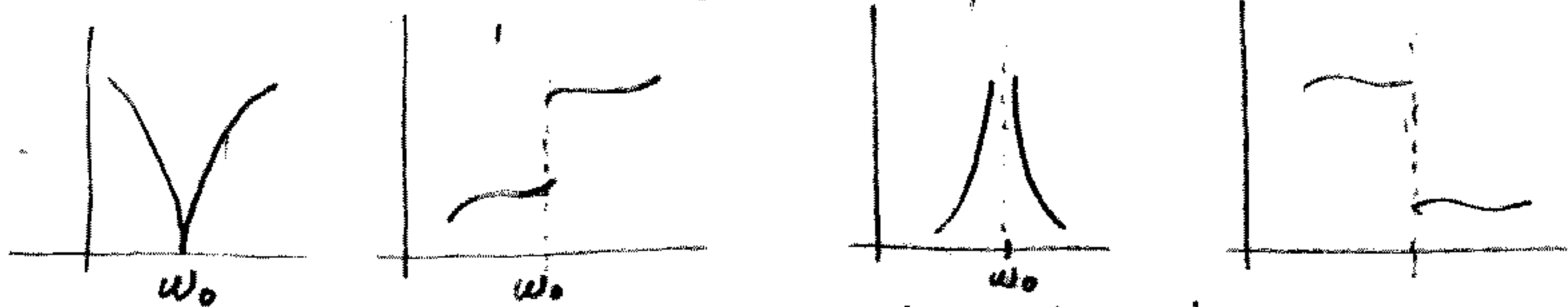
- 2/ Polo en el origen
 - bloque altas frec
 - Aporta $-\frac{\pi}{2} \forall \omega$



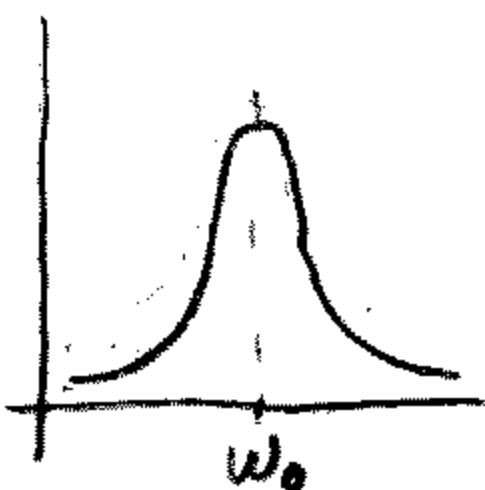
- 3/ Ceros complejos conjugados

- En el eje jw

- 4/ Polos complejos conjugados



- 5/ Polos complejos conjugados con parte real negative



$$H(s) = \frac{N(s)}{D(s)[s^2 + 2p\omega_0 s + \omega_0^2]}$$

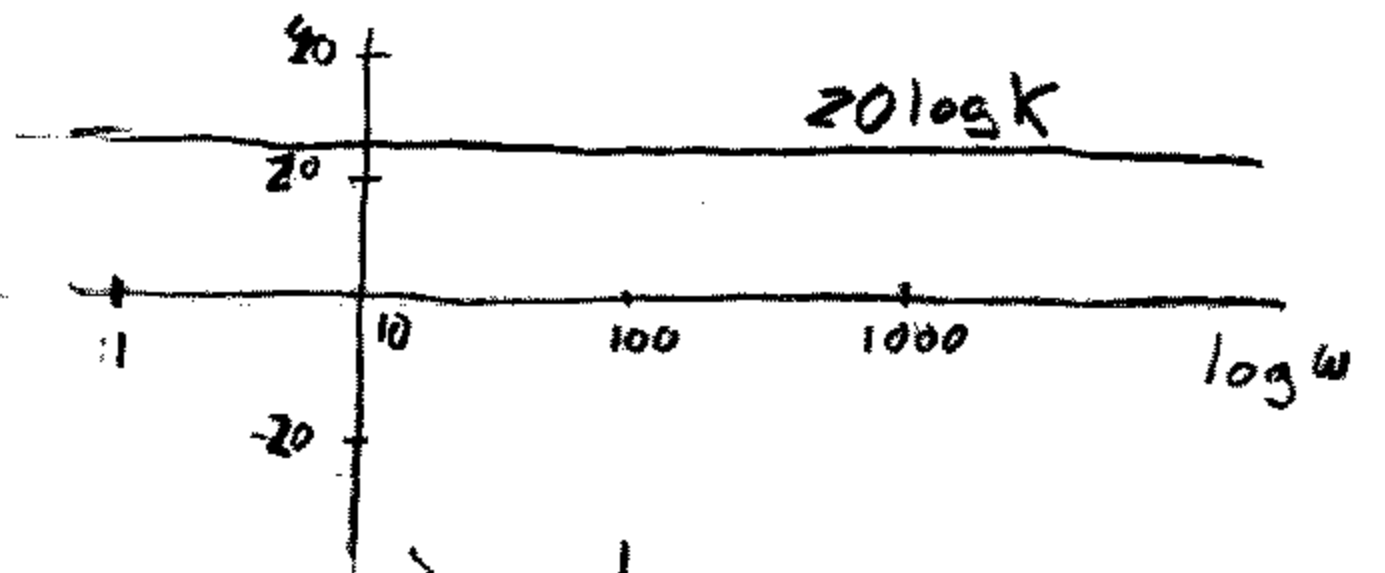
$p \ll 1$

- pico en ω_0
- $BW = 2p\omega_0$
- $Q = \frac{1}{2p}$

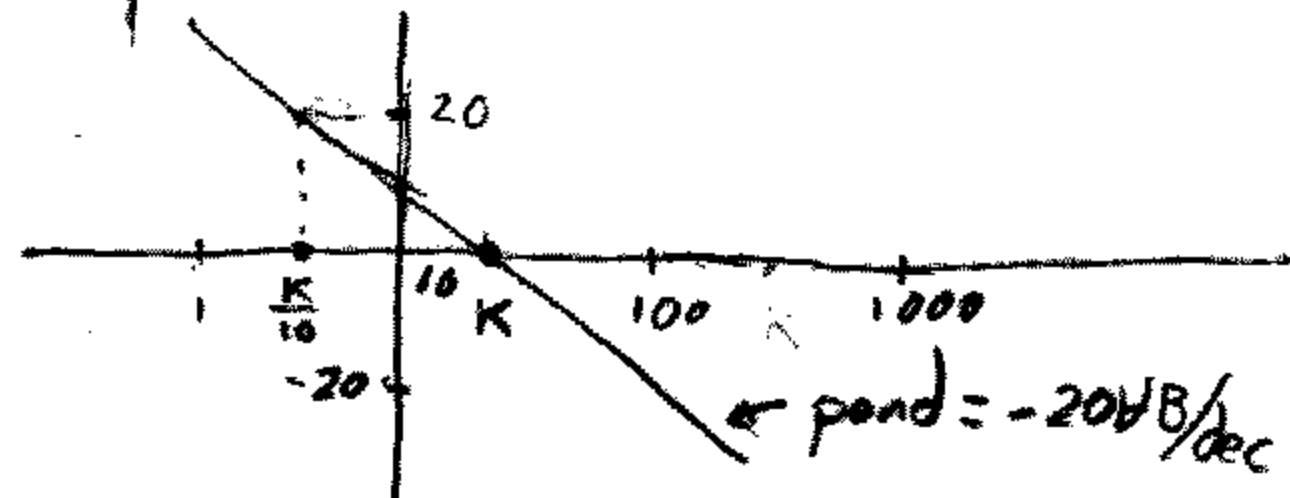
TRAZADOS DE BODE

- Trazados elementales

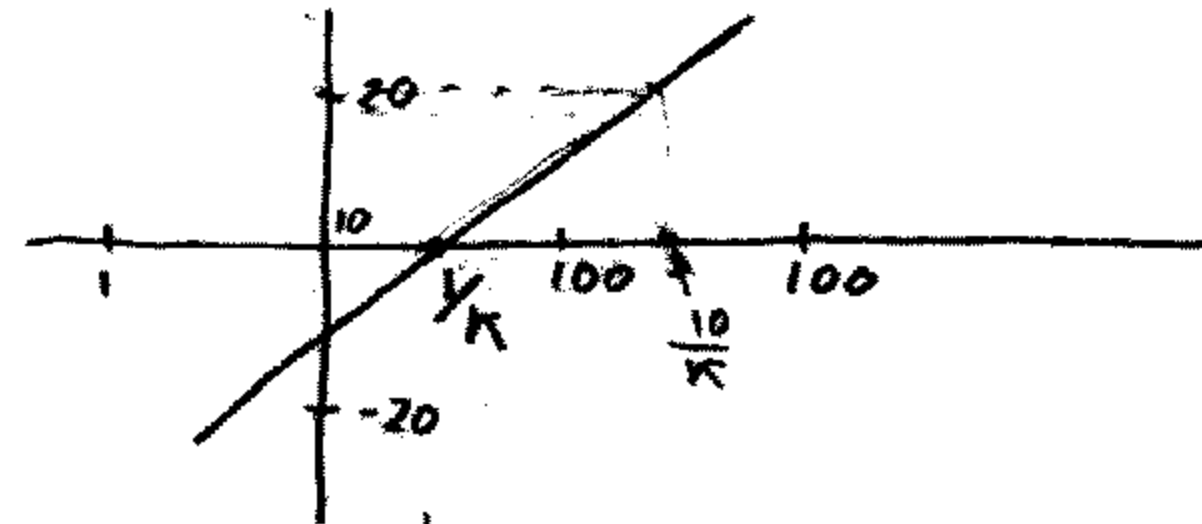
* $H(s) = k \rightarrow 20 \log |H(j\omega)| = 20 \log k$
 $\arg(H(j\omega)) = \begin{cases} 0 \\ \pi \end{cases}$



* $H(s) = \frac{k}{s} \rightarrow 20 \log |H(j\omega)| = 20 \log k - 20 \log \omega$
 $\arg H(j\omega) = -\frac{\pi}{2}$



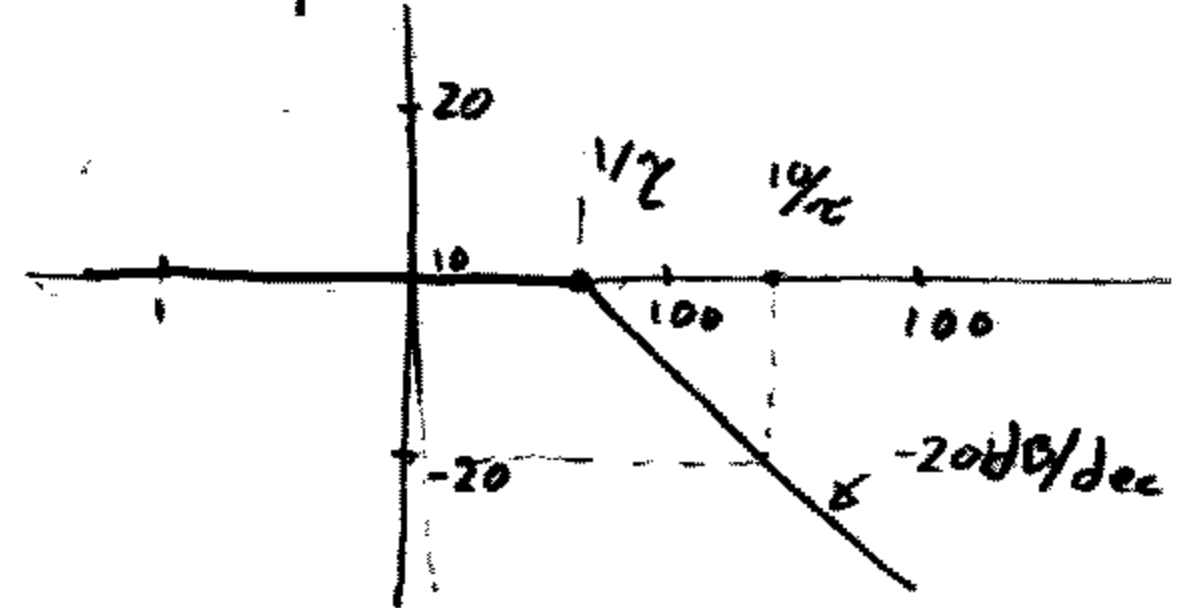
* $H(s) = ks \rightarrow 20 \log |H(j\omega)| = 20 \log k + 20 \log \omega$
 $\arg H(j\omega) = \frac{\pi}{2}$



* $H(s) = \frac{1}{\tau s + 1}$

$\omega \ll \omega_0 \quad G_{dB} = 0$
 $\omega \gg \omega_0 \quad G_{dB} = -20 \log(\tau \omega)$
 Error máximo (en puntos) = -3 dB

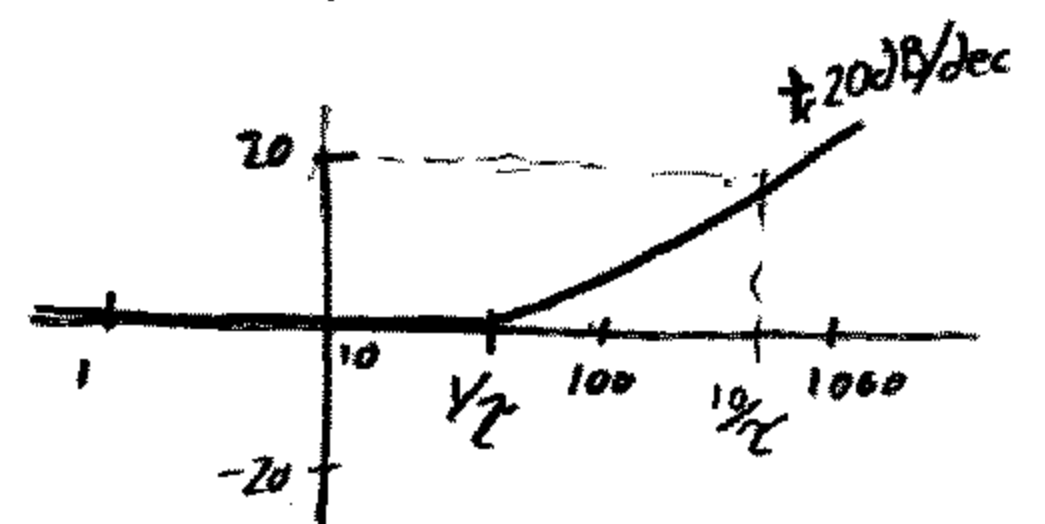
Arg H(jω)
0
-π/2
-π/4



* $H(s) = \tau s + 1$

$\omega \ll \omega_0 \quad G_{dB} = 0$
 $\omega \gg \omega_0 \quad G_{dB} = 20 \log(\tau \omega)$
 Error máx = +3 dB

Arg H(jω)
0
π/2
π/4

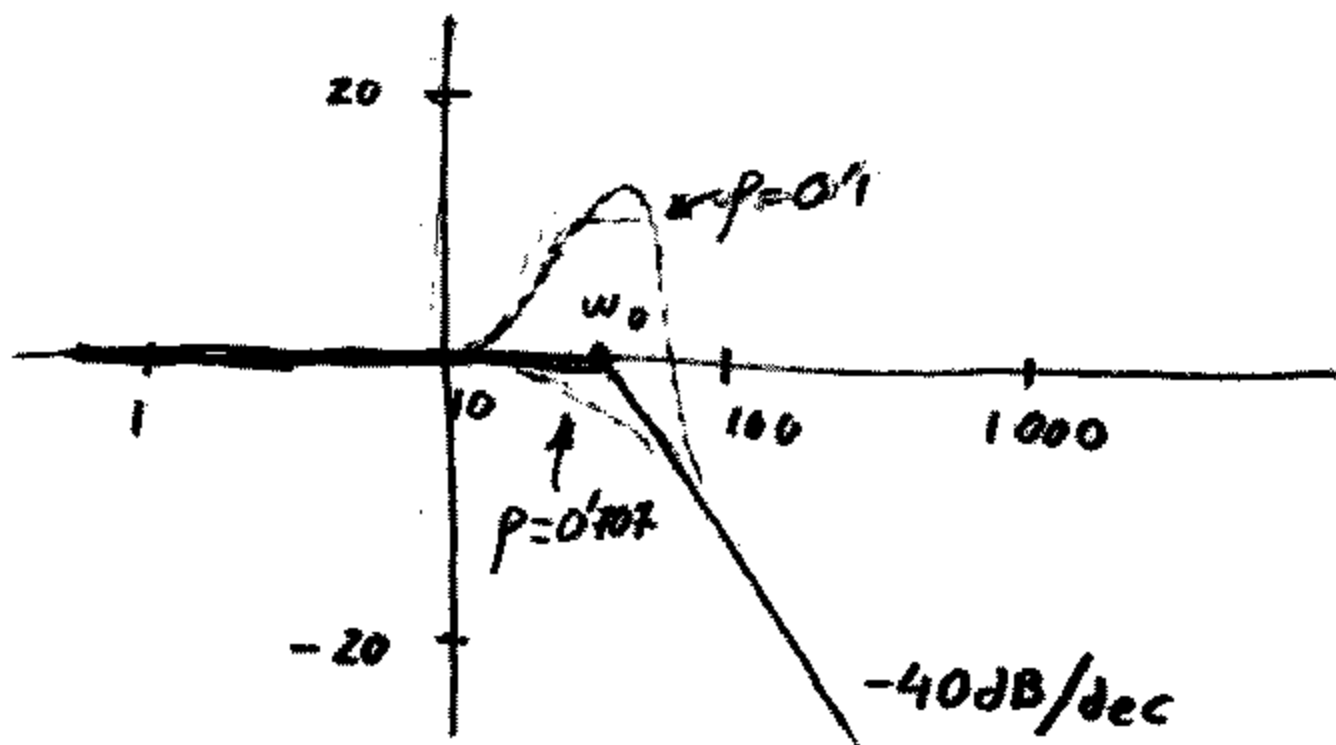


* $H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$ con $0 < \zeta < 1$

$\omega \ll \omega_0 \rightarrow 20 \log |H(j\omega)| = 0$
 $\omega \gg \omega_0 \rightarrow \dots = 40 \log \omega_0 - 40 \log \omega$
 Error en los puntos $\Rightarrow G_{dB\omega_0} = 20 \log \frac{1}{2\zeta}$

ζ	G _{dBω₀} (corrección)
0.9	-5.1 dB
0.8	-4
0.707	-3
0.5	0
0.2	8
0.1	14
0	∞

centro = ω₀



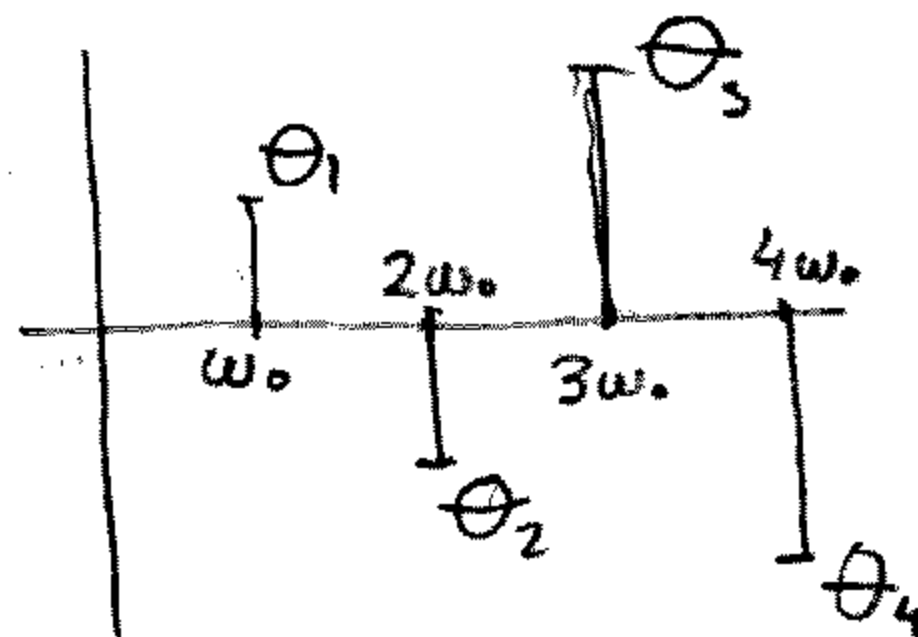
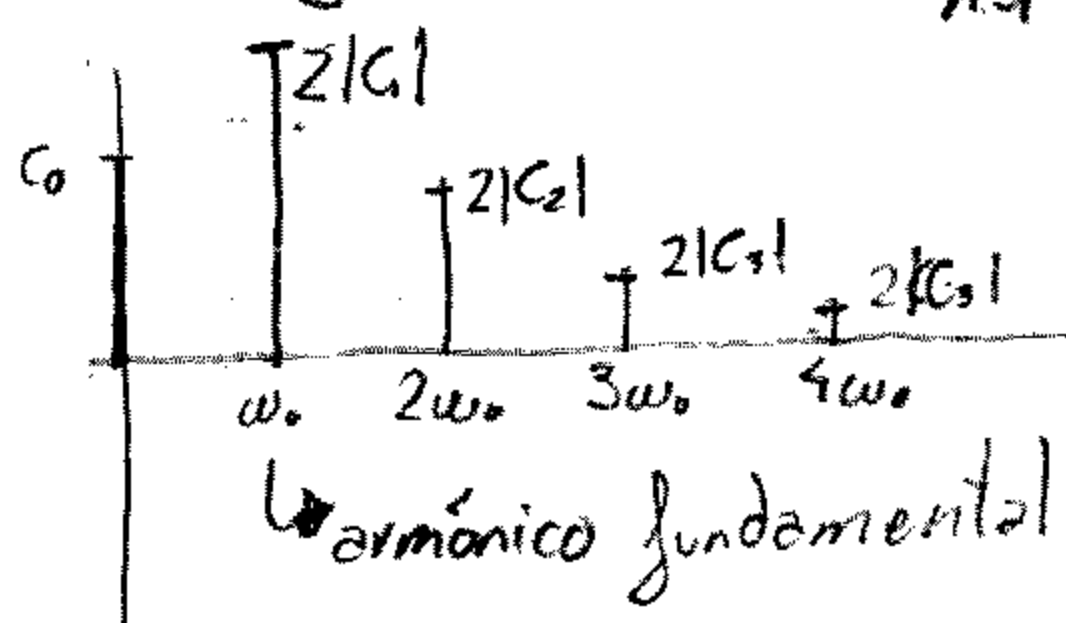
DESARROLLO EN SERIE DE FOURIER

$$v(t) = v(t + nT_0) = C_0 + 2|C_1| \cos(\omega_0 t + \theta_1) + 2|C_2| \cos(2\omega_0 t + \theta_2) + \dots + 2|C_n| \cos(n\omega_0 t + \theta_n)$$

$$C_0 = \frac{1}{T_0} \int_{T_0} v(t) dt ; \quad C_n = \frac{1}{T_0} \int_{T_0} v(t) e^{-jn\omega_0 t} dt = |C_n| e^{j\theta_n}$$

- Representación espectral de senoides

$$v_g(t) = v_g(t + nT_0) = C_0 + \sum_{n=1}^{\infty} 2|C_n| \cos(n\omega_0 t + \theta_n)$$



Desarrollo en serie de Fourier de una SEÑAL CUADRADA

n	0	1	2	3	4	5	...
C_n	V_m/2	V_m/n	0	V_m/3n	0	V_m/5n	...
arg(C_n)	0	-pi/2	0	-pi/2	0	-pi/2	...

* PROPIEDADES DEL DESARROLLO EN SERIE DE FOURIER

$$v(t) \longleftrightarrow C_0 + \sum_{n=1}^{\infty} 2|C_n| \cos(n\omega_0 t + \theta_n)$$

$$\textcircled{1} v(t) + V_{DC} \longleftrightarrow C_0 + V_{DC} + \sum_{n=1}^{\infty} 2|C_n| \cos(n\omega_0 t + \theta_n)$$

$$\textcircled{2} v(t + \tau) \longleftrightarrow C_0 + \sum_{n=1}^{\infty} 2|C_n| \cos(n\omega_0 t + \theta'_n) \quad \theta'_n = \theta_n - n\omega_0 \tau$$

$$\textcircled{3} dv/dt \longleftrightarrow C'_0 + \sum_{n=1}^{\infty} 2|C'_n| \cos(n\omega_0 t + \theta_n) \quad C'_n = jn\omega_0 C_n$$

$$\int_+ v(t) dt \longleftrightarrow \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad C'_n = \frac{C_n}{jn\omega_0}$$

Desarrollo de Fourier de una señal triangular

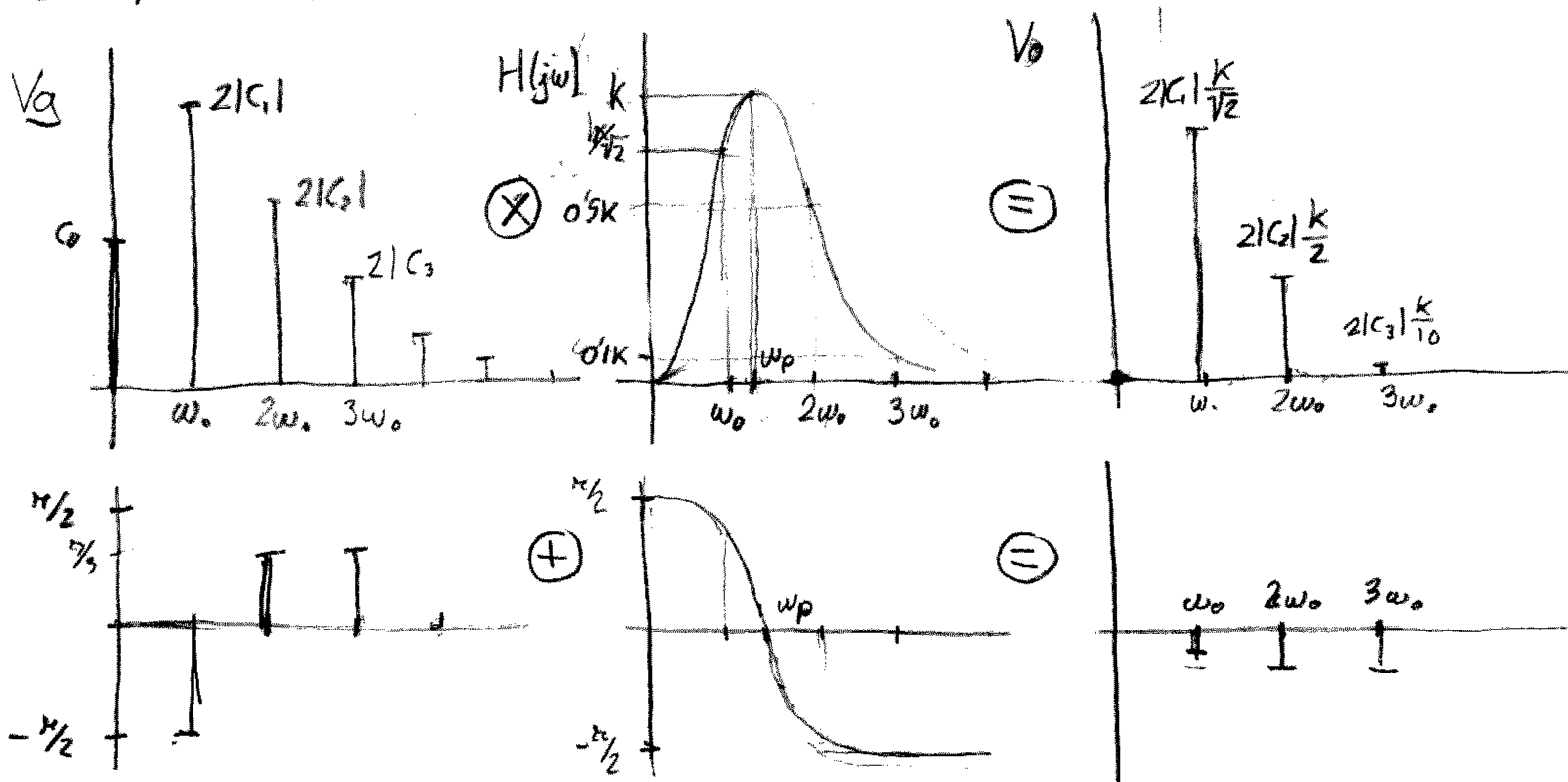
$$v(t) = \frac{8V_m}{\pi^2} \cos(\omega_0 t - \pi) + \frac{8V_m}{9\pi^2} \cos(3\omega_0 t - \pi) + \frac{8V_m}{25\pi^2} \cos(5\omega_0 t - \pi) \dots$$

CIRCUITOS EXCITADOS CON SEÑALES PERIÓDICAS

$$V_o(t) = C_0 |H(0)| e^{j \arg H(0)} + 2|C_1| |H(j\omega_0)| \cos(\omega_0 t + \theta_1 + \arg H(j\omega_0)) + \dots$$

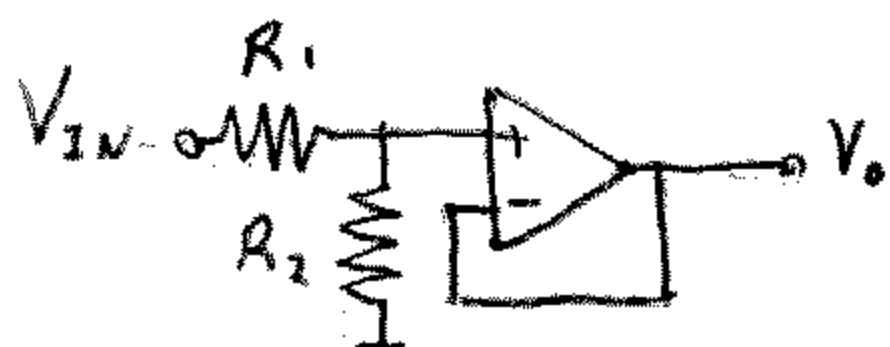
$$+ 2|C_2| |H(j2\omega_0)| \cos(2\omega_0 t + \theta_2 + \arg H(j2\omega_0)) + \dots$$

Interpretación:



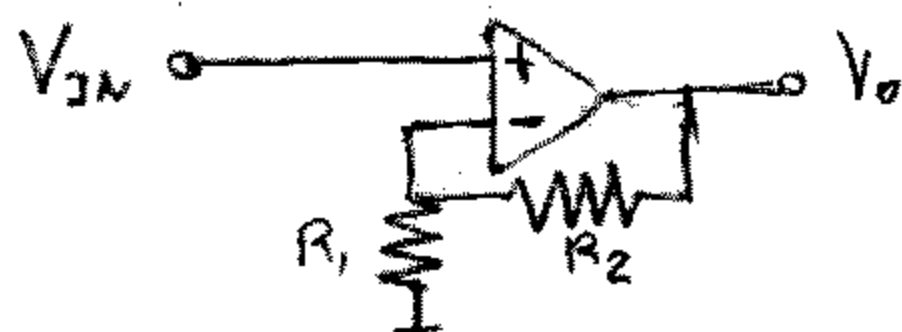
- DISEÑO DE CIRCUITOS ESPECIFICADOS POR \$H(S)\$

* \$H(S) = K\$
\$0 < K \le 1\$



$$K = \frac{R_2}{R_2 + R_1}$$

* \$H(S) = K\$
\$K > 1\$



$$K = 1 + \frac{R_2}{R_1}$$

* \$H(S) = \frac{K}{S}\$



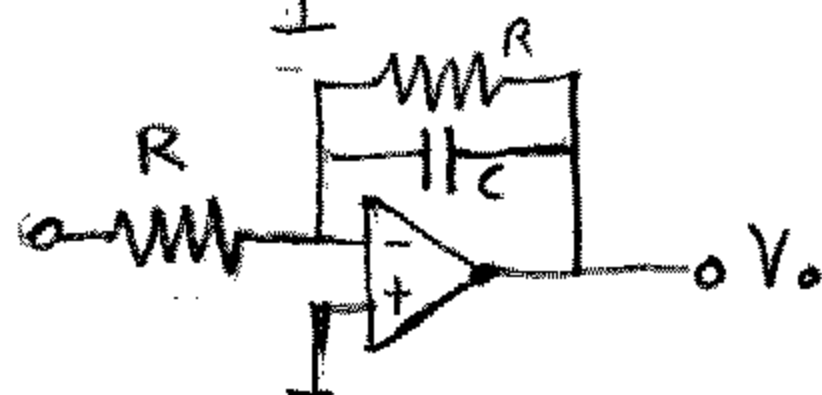
$$K = \frac{1}{RC}$$

* \$H(S) = KS\$



$$K = RC$$

* \$H(S) = \frac{1}{\tau S + 1}\$



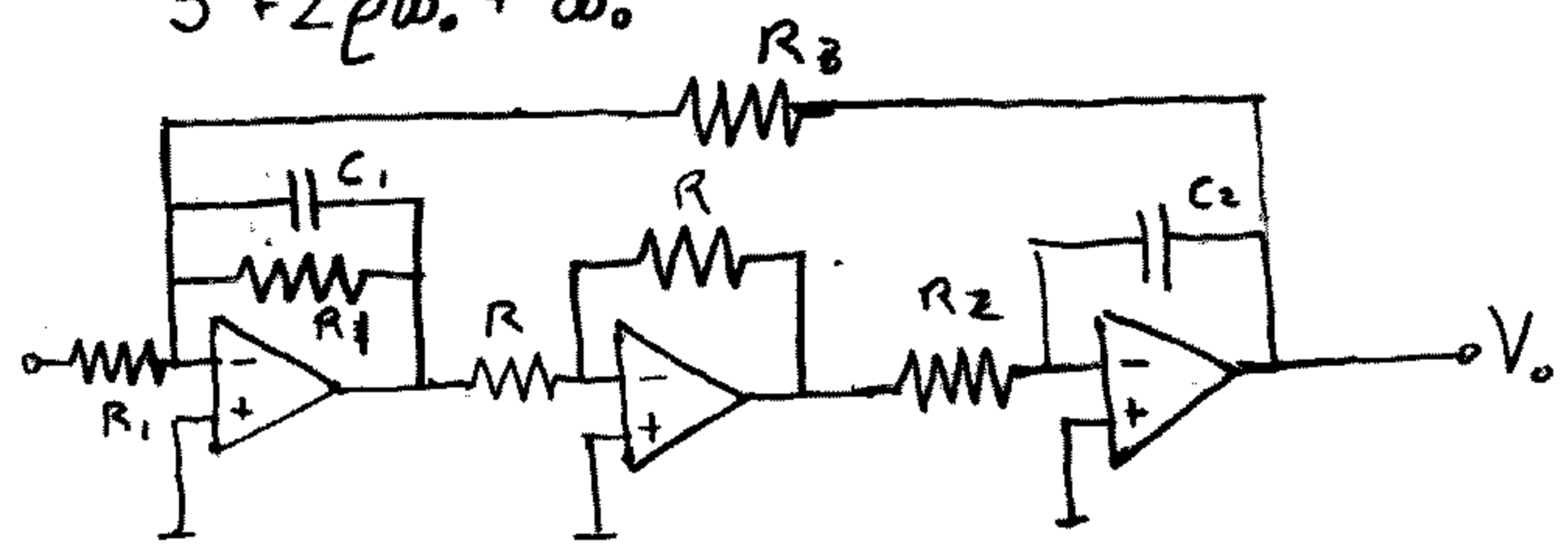
$$\tau = RC$$

* \$H(S) = \tau S + 1\$



$$\tau = RC$$

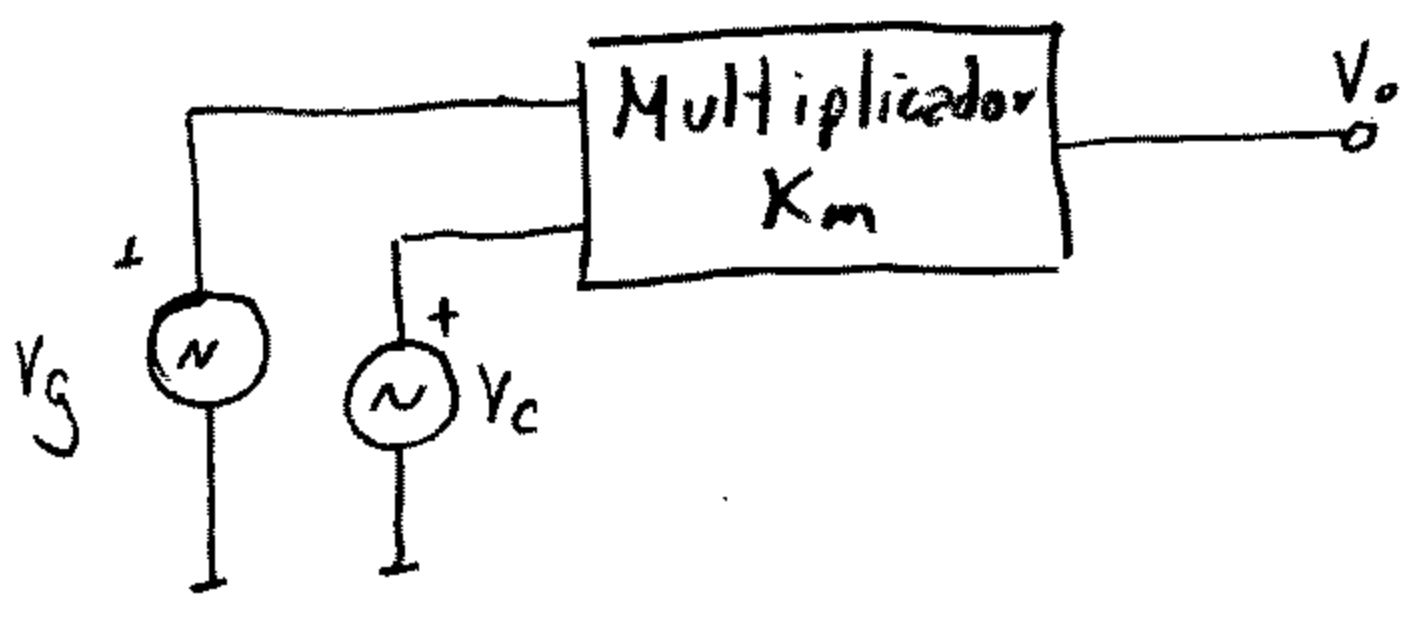
* $H(s) = \frac{\omega^0}{s^2 + 2\rho\omega_0 + \omega_0^2}$



$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_2 R_3}}$

$\rho = \frac{\sqrt{R_2 R_3}}{2 R_1}$

- DESPLAZAMIENTO DE ESPECTROS



$V_g = V_m \cos(\omega_m t)$
 $V_c = V_c \cos(\omega_c t)$

$V_o = K_m V_m V_c \cos(\omega_c t) \cos(\omega_m t) = K_m V_m V_c [\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t]$

