ONES ACÚSTIQUES i ULTRASONS

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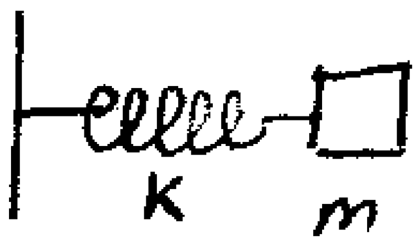
T2: FONAMENTS DE VIBRACIONS i ONES

◦ Oscil·lador harmònic



$$\boxed{m x'' + K x = 0}$$

sol.: $\boxed{x(t) = A\cos(\omega_0 t + \varphi_0)}$ $\boxed{\omega_0 = \sqrt{\frac{K}{m}}}$

$$\boxed{\begin{array}{l} U = \frac{1}{2} K x^2 \\ E_K = \frac{1}{2} m v^2 \end{array}}$$

$$\boxed{E = U + E_K}$$

◦ Oscil·lador esmorteït

$$\boxed{m x'' + b x' + K x = 0}$$

$\boxed{\tau = \frac{m}{b} = \frac{1}{2\alpha}}$ $\boxed{\Delta\omega = \omega_0 - \omega'}$

sol.: $\boxed{x(t) = A_0 e^{-\alpha t}\cos(\omega' t + \varphi_0)}$ $\boxed{\alpha = \frac{b}{2m}}$ $\boxed{\omega'^2 = \omega_0^2 - \alpha^2}$

◦ Moviment forçat

$$\boxed{m x'' + b x' + K x = F(t)}$$

$F(t) = F_0 e^{j\omega t} \rightarrow x_e(t) = \vec{A} e^{j\omega t} \ldots)$

$$\boxed{Z_m = \frac{F}{v} = b + j(\omega m - K/\omega)}$$

$\omega = \omega_0 \Rightarrow \theta_v = 0,\ Z_m = b$

$\boxed{Q = \omega_0 \tau \simeq \frac{\omega_0}{\Delta\omega}}$ $\boxed{P_0 = \frac{F_0^2}{2|Z_m|}\cos(\theta_v)}$

– ONES PROGRESSIVES i ESTACIONÀRIES

$\boxed{\mu \frac{\partial^2 \psi}{\partial t^2} = T \frac{\partial^2 \psi}{\partial x^2}}$ $\boxed{c = \sqrt{\frac{T}{\mu}}}$

◦ Ones harmòniques

$\boxed{\psi(t,x) = A\cos(\omega t - kx + \varphi_0)}$ $\boxed{c = \frac{\omega}{K} = f\lambda}$

◦ Energia d'una ona progressiva

$\boxed{\eta_K = \frac{1}{2}\mu\left(\frac{\partial\psi}{\partial t}\right)^2}$ $\boxed{\eta_U = \frac{1}{2} T\left(\frac{\partial\psi}{\partial x}\right)^2}$ $\boxed{P = c\eta}$

$\boxed{F = -T\frac{\partial\psi}{\partial x}}$ $\boxed{Z = c\mu}$

◦ Dispersió

$\boxed{v_f = \overline{\omega}/\overline{K}}$ $\boxed{v_g = \Delta\omega/\Delta K}$

◦ Ones estacionàries

$$\boxed{\psi(t,x) = A\sin(Kx)\sin(\omega t)}$$

T3: PROPAGACIÓ D'ONES EN UN FLUID

$\boxed{dW = P \cdot dV}$ $\boxed{\rho = \frac{m}{V}}$ $m = nM$

Compressió Isotèrmica ⇒ a temp. cte
" " Adiabàtica ⇒ sense intercanvi de calor

Dilatació: $\boxed{\frac{\partial \rho}{\rho} = -\frac{\partial V}{V} = -S}$

Compressibilitat: $\boxed{B = -V\frac{\partial P}{\partial V} = -\frac{\partial P}{S}}$

$\boxed{dQ = dU = c_v\, dT}$ Calor esp. a volum cte.

$\boxed{dQ = dU + P\,dV = c_p\, dT}$ Calor espec. a pressió cte.

$\boxed{\gamma = c_p/c_v = B_{adiab}/B_{isot}}$

$\boxed{PV = nRT}$ $N = nN_a$ $K = R/N_a$

$k = 1'38 \cdot 10^{-23}\ J/K$
$R = 8'3\ J/K \cdot mol$
$N_a = 6'022 \cdot 10^{23}$
$1\,atm = 101'23 \cdot 10^3\ Pa$

- Teoria cinètica

$\boxed{\lambda = \tau\, v_{rms}}$ $\boxed{\langle E_K \rangle = 3\left(\frac{1}{2}KT\right) = \frac{1}{2} M v_{rms}^2}$ d'una molècula

$\boxed{U = f N_a\left(\frac{1}{2}KT\right)}$

gasos monoatòmics $f = 3$ $\gamma = 1'67$
gasos diatòmics $f = 5$ $\gamma = 1'40$

$\boxed{c_v = 1/2\, f R}$ $\boxed{c_p = c_v + R}$ d'un mol

$\boxed{B_{isot} = P}$ $\boxed{B_{adiab} = \gamma P}$

- Recorregut lliure mig

$$\lambda_{rms} = \frac{Volum}{A} = \frac{Volum}{\pi(2r)^2}$$

– PROPAGACIÓ EN UNA DIMENSIÓ (cond. adiabàtiques)

$\boxed{S = \frac{\Delta V}{V} = \frac{\partial u}{\partial x}}$ $\boxed{p = -BS}$ $\rho\frac{\partial^2 u}{\partial t^2} = B\frac{\partial^2 u}{\partial x^2}$

$\boxed{c = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{dP}{d\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma\frac{RT}{M}}}$ $\boxed{Z = \frac{p}{v} = \rho c}$ $\frac{\partial P}{P} + \frac{\partial V}{V} = \frac{\partial T}{T}$

$\boxed{\eta_K = \frac{1}{2}\rho v^2}$ $\boxed{\eta_U = \frac{1}{2}\frac{p^2}{\rho c^2}}$ $\boxed{I = \eta c}$ $\langle I \rangle = \frac{1}{2}\frac{P^2}{Z}$

$NI = 10\log\frac{I}{I_{ref}}$

– PROPAGACIÓ EN 3D

$$S = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$$

- **Atenuació**

$$\boxed{1\,Np/m = 8'686\, dB/m}$$

$$\boxed{-p = BS + \eta \frac{\partial S}{\partial t}}$$ η: coef. viscositat

$$\boxed{\tau = \frac{\eta}{B}} \quad \boxed{\alpha = \frac{\omega^2 \tau}{2 c_0}} \quad \boxed{c = c_0\left(1 + \frac{3}{8}\omega^2 \tau^2\right)}$$

- **Pressió de radiació**

$$\boxed{F = \frac{IA}{c}} \quad \boxed{P_{rad} = \frac{F}{A} = \frac{I}{c}} = \eta$$

- **Efecte Doppler**

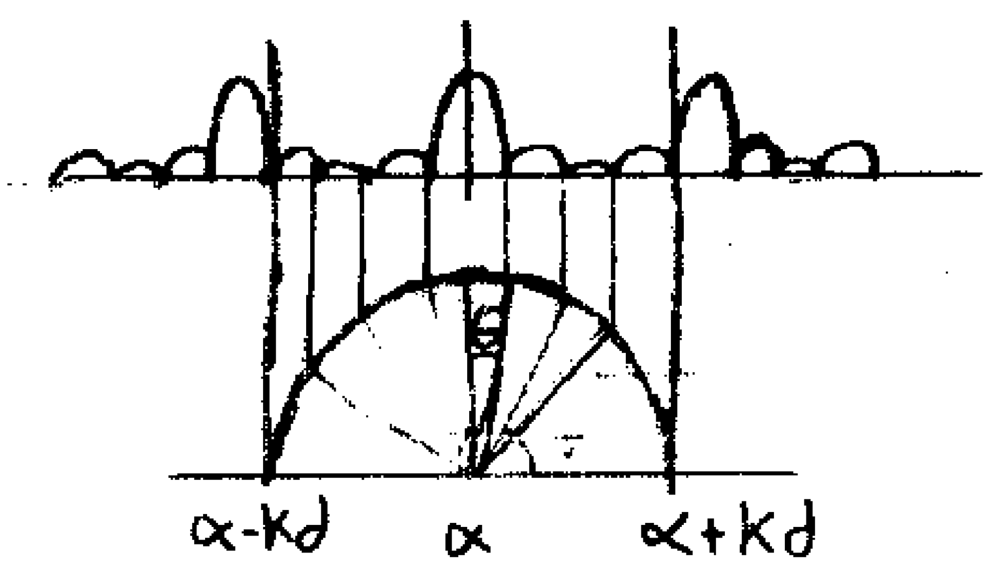
$$\boxed{f = f_0 \frac{c - V_R}{c - V_E}}$$

T4: RADIACIÓ. FENÒMENS DE DIFRACCIÓ

- **Array de N focus igualment espaiats**

$$\boxed{|p| = A \left| \frac{\sin\left(\frac{N}{2}\Delta\right)}{\sin\left(\frac{1}{2}\Delta\right)} \right|}$$

$$\boxed{\Delta = Kd \sin\theta + \alpha}$$



- **Focus lineal**

$$\boxed{|p| = A \left| \frac{\sin \Delta/2}{\Delta/2} \right|}$$

- **Font puntual**

$$\boxed{Z = \rho c = \frac{j\omega\rho r}{1 + jKr}} \quad \langle I \rangle = \frac{p^2}{2\rho c} \quad Pot = \langle I \rangle \cdot A$$

$A_{esf} = 4\pi r^2$

- **Pistó circular** (radi a)

$Jinc(x) = \frac{2J_1(x)}{x}$

aprox $z > z_F$

$$\boxed{p = \frac{j\rho v_0 K a^2}{2r} c \cdot Jinc(Ka\sin\theta)\, e^{j(\omega t - Kr)}}$$

$Jinc(x) = 0 \Leftrightarrow x \simeq (n + 1/4)\pi$

en el eje $z < z_F$ aprox.

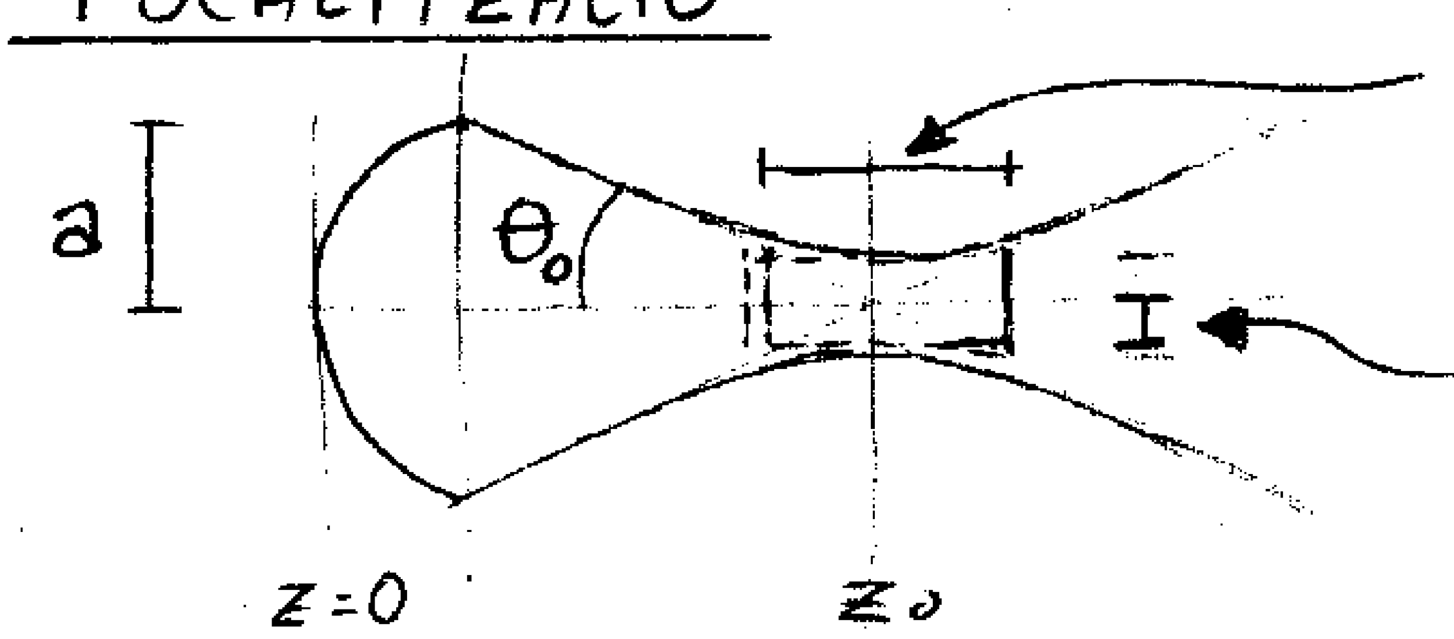
$$\boxed{|p| = \rho c\, v_0 \sqrt{2 - 2\cos(K\beta)}} \quad \boxed{\beta = \sqrt{z^2 + a^2} - z} \quad \boxed{z_F \simeq \frac{a^2}{\lambda}}$$

$$\boxed{R_n = z + n\frac{\lambda}{2} = \sqrt{z^2 + \sigma_n^2}}$$

σ_n: Radi

La integral sobre dues zones consecutives és $\emptyset$

- Focalització



$$\boxed{d_z = \frac{1'22\,\lambda}{\sin^2\theta_0}}$$

$$\boxed{r_0 = \frac{0'61\lambda}{\sin\theta_0}}$$

$$\sin\theta_0 = \frac{a}{z_0}$$

T5: Acústica en sòlids

$$S_{ii} = \frac{\partial u_i}{\partial x_i} = \frac{\Delta x}{x} \quad [S_{ij}] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \qquad [S_p] = (S_{11}, S_{22}, S_{33}, 2S_{23}, 2S_{13}, 2S_{12})$$

o canvi s. referència $\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\quad S'_{ij} = \alpha_{ik}\alpha_{jl} S_{kl} \quad \bar{\bar{S}}' = \bar{\bar{\alpha}}\,\bar{\bar{S}}\,\bar{\bar{\alpha}}^{-1}$

o diagonalització $\quad |\bar{\bar{S}} - \lambda \bar{\bar{I}}| \overset{!!}{=} 0 \quad (\bar{\bar{S}} - \lambda_i \bar{\bar{I}}) \cdot \overline{vep_i} \overset{!!}{=} \overline{vep_i} \quad \bar{\bar{\alpha}} = \begin{bmatrix} vep_1 \\ vep_2 \\ vep_3 \end{bmatrix}$

- Esforços

Sobre una superfície $\quad d\vec{F} = \bar{\bar{T}} \cdot d\vec{S} \quad \boxed{dF_i = T_{ij}\, dS_j}$

Sobre una sup. tancada $\quad \boxed{\Delta F_i = \frac{\partial T_{ij}}{\partial x_j} \Delta V}$

$$[T_{ij}] = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{12} & T_{22} & T_{23} \\ T_{13} & T_{23} & T_{33} \end{bmatrix} \qquad [T_p] = (T_{11}, T_{22}, T_{33}, T_{23}, T_{13}, T_{12})$$

$$\boxed{dW = T_p\, dS_p} \qquad T_p = \frac{\partial U}{\partial S_p}$$

- Elasticitat

$$\bar{\bar{s}}\,\bar{T} = \bar{S} \Rightarrow s_{pq} T_q = S_p \qquad \bar{\bar{c}}\,\bar{S} = \bar{T} \Rightarrow c_{pq} S_q = T_p$$

$$\bar{\bar{c}}^{-1} = \bar{\bar{s}} = \begin{bmatrix} s_{11} & s_{12} & s_{12} & & & \\ s_{12} & s_{11} & s_{12} & & & \\ s_{12} & s_{12} & s_{11} & & & \\ & & & s_{66} & & \\ & & & & s_{66} & \\ & & & & & s_{66} \end{bmatrix}$$

$$\boxed{s_{11} = \frac{1}{E}} \quad \boxed{s_{12} = -\frac{\sigma}{E}} \quad \boxed{s_{66} = \frac{2(1+\sigma)}{E}}$$

$$\boxed{c_{11} = \lambda + 2\mu} \quad \boxed{c_{12} = \lambda} \quad \boxed{c_{66} = \mu}$$

$$\boxed{\mu = \frac{E}{2(1+\sigma)}} \quad \boxed{\frac{\lambda}{\lambda + 2\mu} = \frac{\sigma}{1-\sigma}}$$

- Propagació d'ones

ona longitudinal $\quad \boxed{V_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}} \qquad V_L = \frac{\omega}{K_L}$

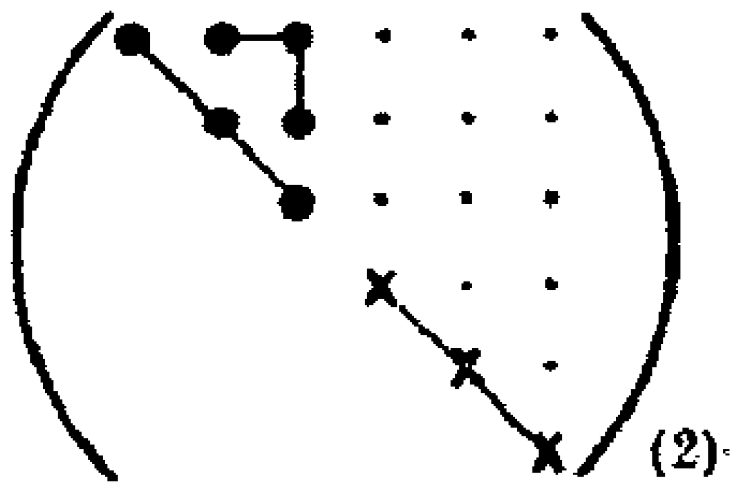
ona transversal $\quad \boxed{V_s = \sqrt{\frac{\mu}{\rho}}} \qquad V_s = \frac{\omega}{K_s} \qquad \lambda = 2\,\text{gross}$

$$\tau = \frac{1}{2\alpha V_s}$$

T6 - ACUSTICA EN CRISTALLS

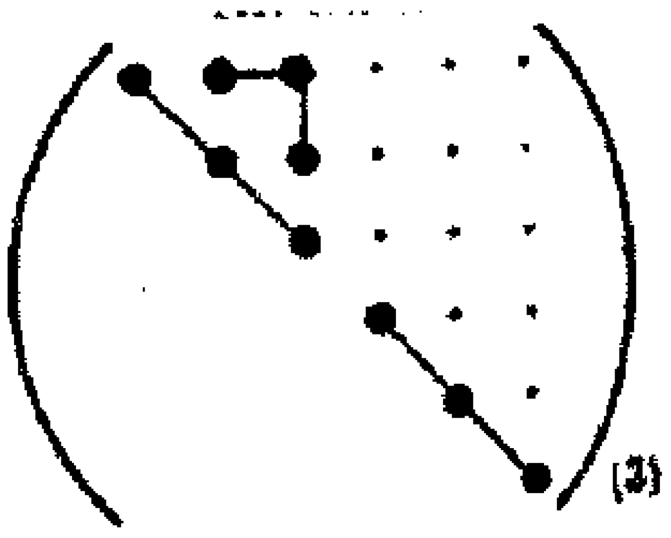
Coeficients elàstics segons el grup de simetria

isòtrop

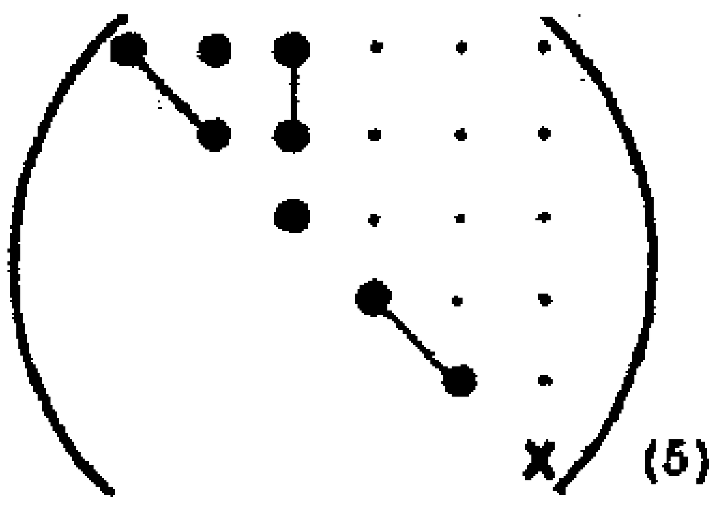


· = 0 ● ≠ 0
●——● iguals
●——○ iguals, signe oposat
⊙ = 2 x ● ◉ = - 2 x ●
$\epsilon = 2(s_{11}-s_{12})$ o bé $\epsilon = \frac{1}{2}(c_{11}-c_{12})$

m3m 432 m3 23 $\underline{4}$3m — **cúbic**

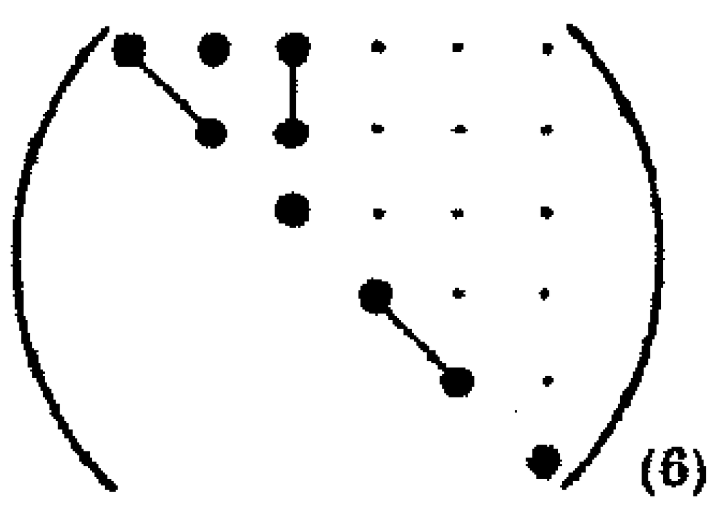


6 $\underline{6}$ 6/m 6/mmm 622 6mm $\underline{6}$m2 — **hexagonal**

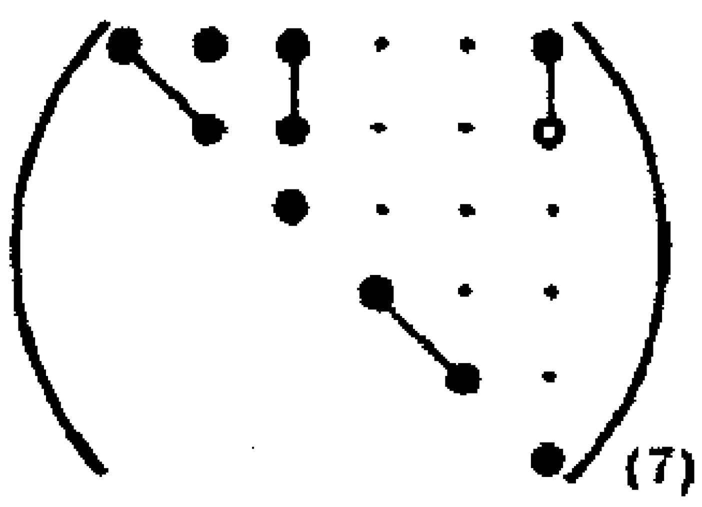


tetragonal

4mm 422 $\underline{4}$2m 4/mmm

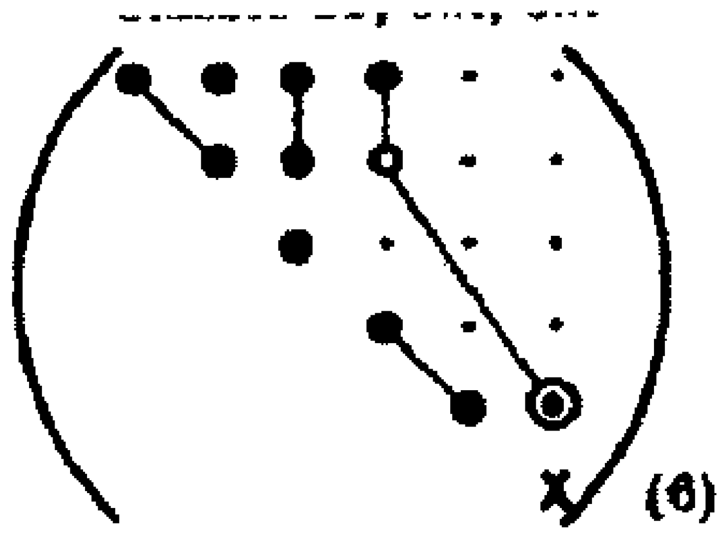


4/m 4 $\underline{4}$

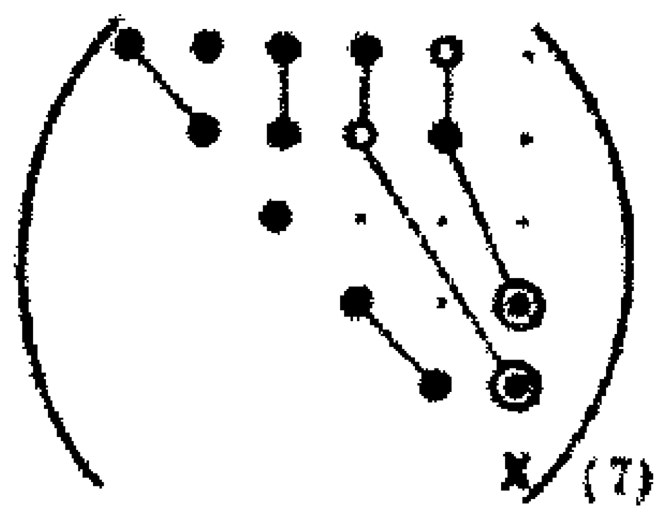


romboèdric

32 3m $\underline{3}$m

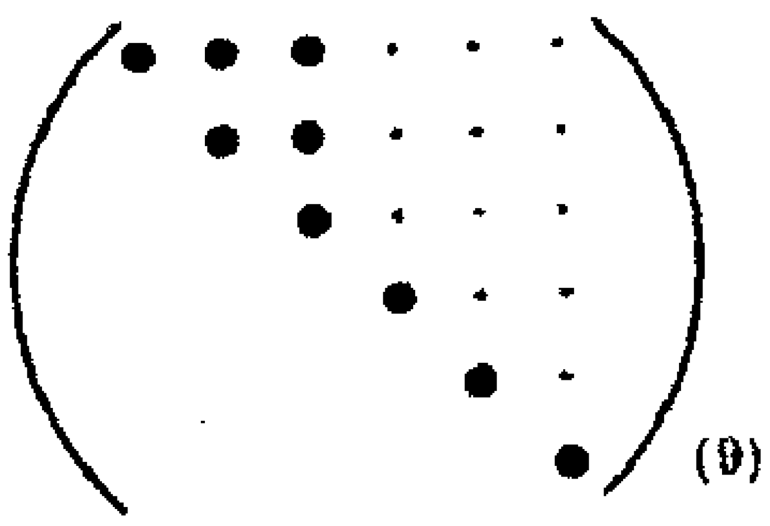


3 $\underline{3}$



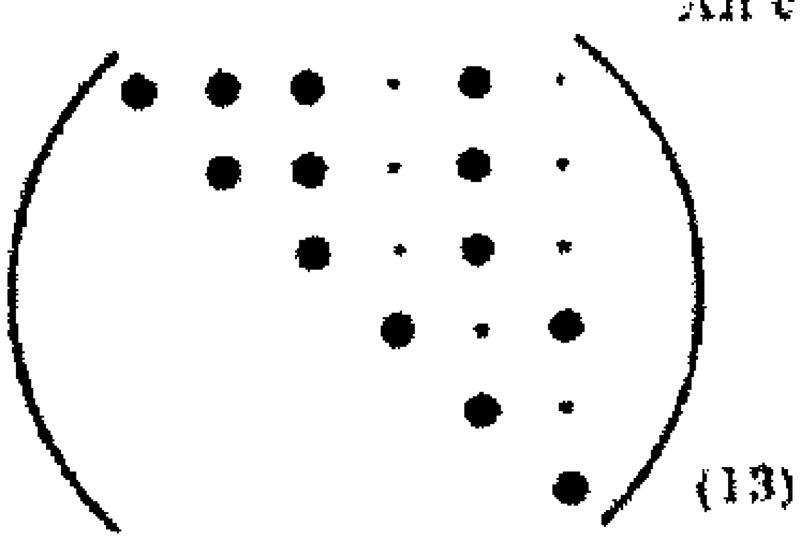
ortorròmbic

mm2 mmm 222



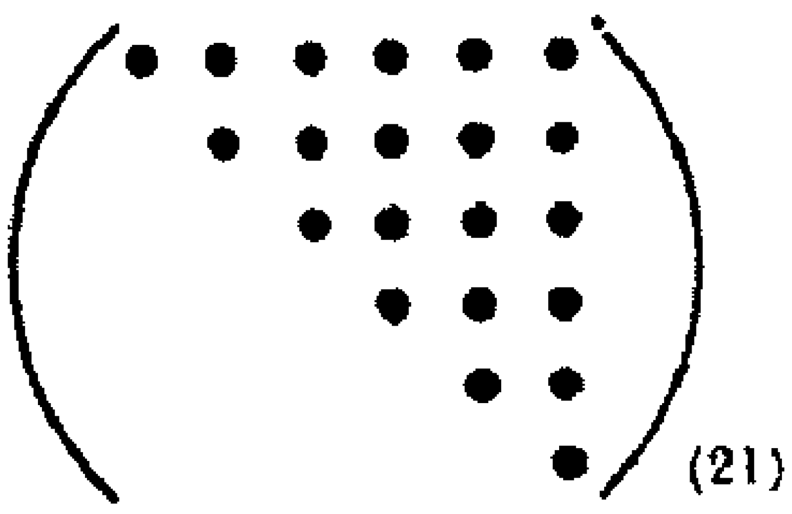
monoclínic

2 m 2/m



triclínic

1 $\underline{1}$

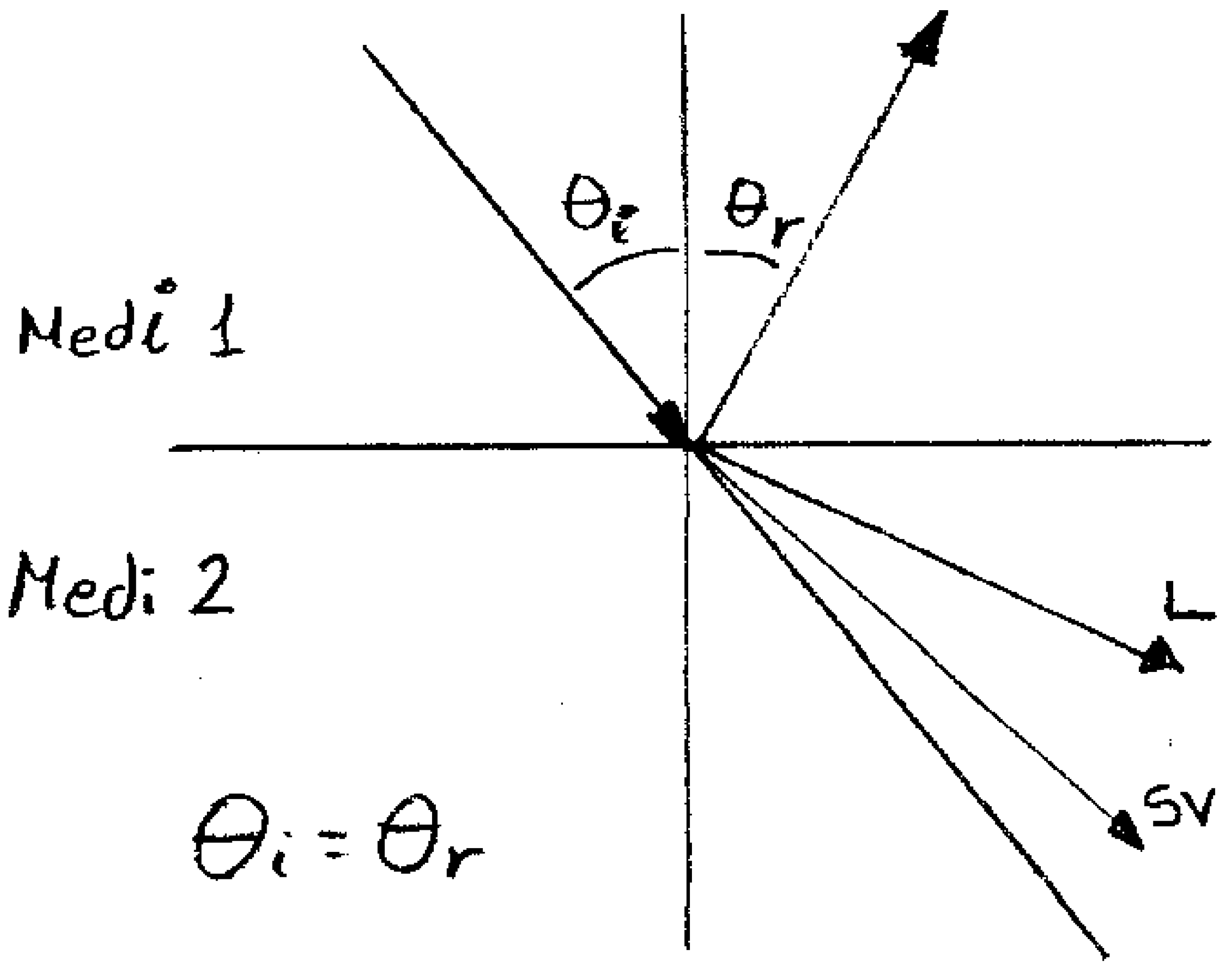


- Matriu de Christoffel

$\boxed{\Gamma_{il} = c_{ijkl} \cdot n_j \cdot n_k} \Rightarrow \Gamma_{il} \cdot u_{0l} = \rho V^2 u_{0i} \Rightarrow \boxed{V_S, V_L = \sqrt{\frac{\Gamma_{il}}{\rho}}}$

En la direcció dels eixos es més fàcil de calcular $\boxed{\Gamma_{il} = c_{innl}}$

17 - Canvis de medi



$\theta_i = \theta_r$

- Coeficients de reflexió i transmissió

$$R_p = \frac{P_r}{P_i} \qquad T_p = \frac{P_t}{P_i} \qquad T_p = 1 + R_p$$

$$R_p = R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad T_p = \frac{\rho_2}{\rho_1} T = \frac{2Z_2}{Z_2 + Z_1}$$

$$SWR = \frac{1 + R_p}{1 - R_p}$$

$$r = \frac{I_r}{I_i} = R_p^2 \qquad t = 1 - r = 1 - R_p^2$$

- Fluid-Fluid → Tot modes L

Llei de Snell $\quad \frac{\sin\theta_i}{V_1} = \frac{\sin\theta_t}{V_2}$

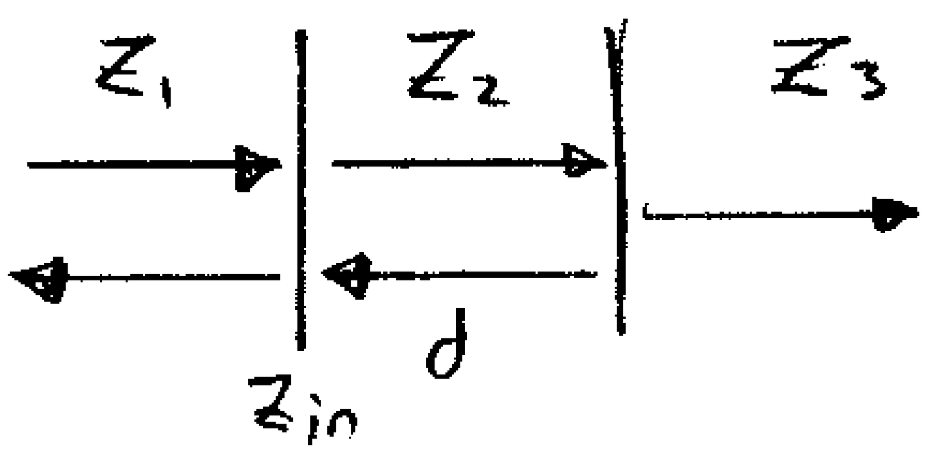
$$Z_1 = \frac{\rho_1 V_1}{\cos\theta_1} \qquad Z_2 = \frac{\rho_2 V_2}{\cos\theta_2}$$

$$p_i + p_r = p_t \qquad I_i = \frac{|P_i|^2}{2\rho_1 V_1} \qquad I_r = \frac{|P_r|^2}{2\rho_1 V_1} \qquad I_t = \frac{|P_t|^2}{2\rho_2 V_2}$$

$$I_i \cos\theta_i = I_r \cos\theta_r + I_t \cos\theta_t$$

Angle crític $\quad \sin\theta_c = \frac{V_1}{V_2} = n$

- Reflexió en una làmina



$$Z_{in} = Z_2 \frac{Z_3 - j Z_2 \, tg\,\varphi}{Z_2 - j Z_3 \, tg\,\varphi} \qquad \varphi = k\ell$$

$$R_p = 0 \Leftrightarrow Z_2 = Z_1 Z_3 \qquad \ell = \lambda/4$$

- Líquid-Sòlid $\quad$ modes transmesos S i L

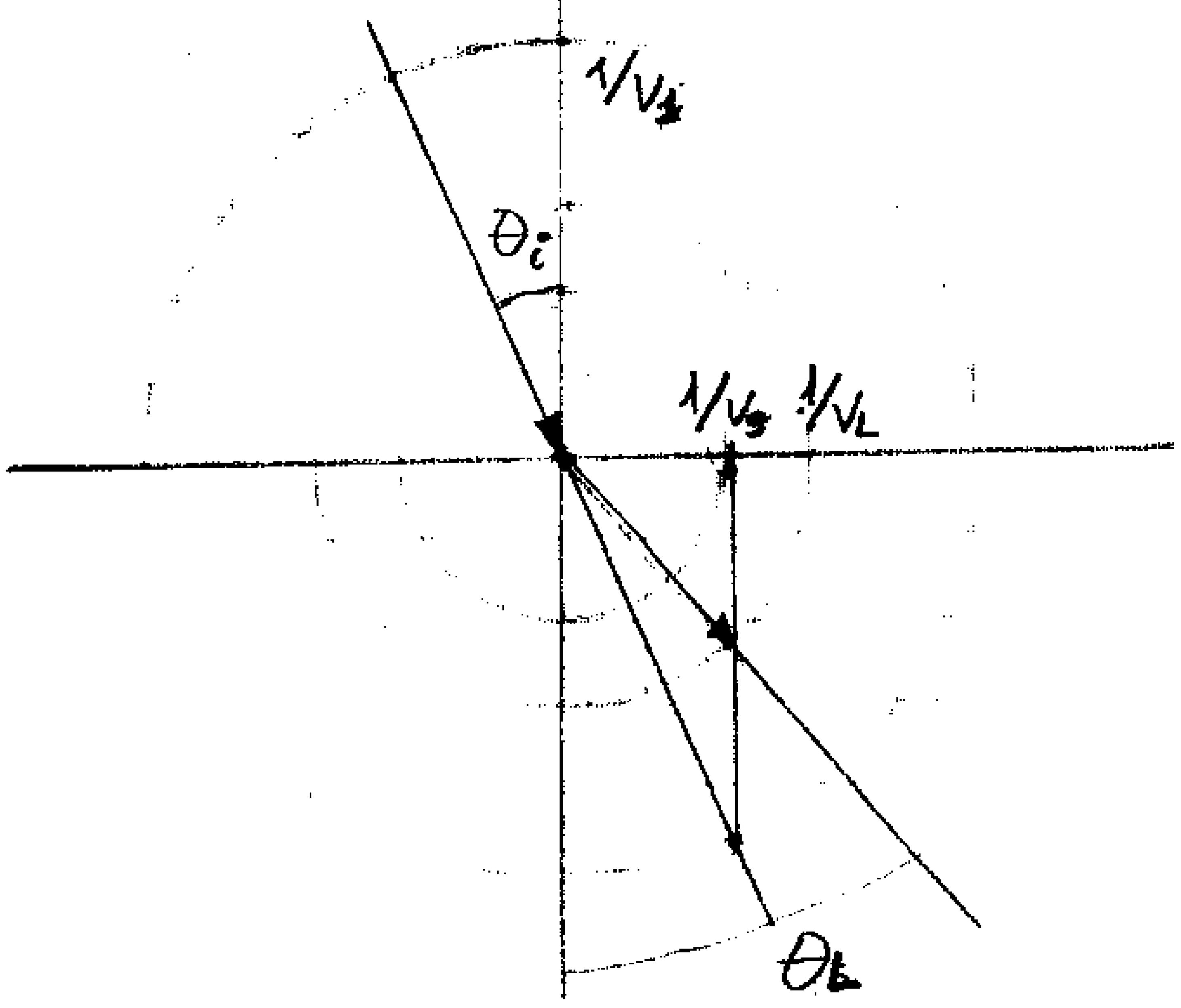
Llei de Snell $\quad \frac{\sin\theta_i}{V_1} = \frac{\sin\theta_L}{V_L} = \frac{\sin\theta_s}{V_s}$

Recordatori ⇒ $\quad V_1^2 = \lambda_1/\rho_1 \qquad V_L^2 = (\lambda_2 + 2\mu_2)/\rho_2 \qquad V_s^2 = \mu_2/\rho_2$

Formula de Fresnel $\quad R = \frac{m\cos\theta_i - \sqrt{n^2 - \sin^2\theta_i}}{m\cos\theta_i + \sqrt{n^2 - \sin^2\theta_i}}$

$$n = V_1/V_2 \qquad m = \rho_2/\rho_1$$

Conversió de modes



18 - ONES SUPERFICIALS i GUIADES

Relació de Rayleigh $\quad \eta = \frac{V}{V_S} \simeq \frac{0'87 + 1'12\sigma}{1+\sigma} \qquad \xi = \frac{V_S}{V_L} = \sqrt{\frac{1-2\sigma}{2(1-\sigma)}}$

$\sigma \equiv$ relació de Poisson

$K_L = \frac{\omega}{V_L} \qquad \gamma_L^2 := \beta^2 - k_L^2$

$K_S = \frac{\omega}{V_S} \qquad \gamma_S^2 = \beta^2 - K_S$

- Si les ones pasen de líquid → Sòlid i generen ones a la sup. sòlid:

$\beta = K_{liq} \sin\theta_i = \frac{\omega}{V} \sin\theta_i$

$\beta = \frac{\omega}{V_R}$

19 - PIEZOELECTRICITAT

$$\begin{bmatrix} [s^E]_{6\times6} & [d]_{3\times6} \\ [d]_{6\times3} & [\varepsilon^T]_{3\times3} \end{bmatrix}_{9\times9} \cdot \begin{bmatrix} [T]_6 \\ [E]_3 \end{bmatrix} = \begin{bmatrix} [S]_6 \\ [D]_3 \end{bmatrix}$$

$$\begin{bmatrix} [c^D] & [-h] \\ [-h] & [\beta^S] \end{bmatrix} \begin{bmatrix} [S]_6 \\ [D]_3 \end{bmatrix} = \begin{bmatrix} [T]_6 \\ [E]_6 \end{bmatrix} \qquad \begin{bmatrix} s^D & g \\ -g & \beta^T \end{bmatrix} \begin{bmatrix} T \\ D \end{bmatrix} = \begin{bmatrix} S \\ E \end{bmatrix}$$

$Q = D \cdot Area = -CV_0$

$V = E \cdot \ell$

$u = S \cdot \ell$

$F = T \cdot Area$

$C = \frac{\varepsilon \cdot Area}{\ell}$

Constant d'acoblament $\quad K_{ij} = \frac{d_{ij}}{\sqrt{\varepsilon^T_{ii} s^E_{jj}}}$

$s^D = s^E (1 - K^2) \qquad \varepsilon^S = \varepsilon^T (1 - K^2)$

$S = s^E T + dE \qquad D = dT + \varepsilon^T E$

cortocircuito $E = 0$
circuito abierto $D = 0$

Compresió - Expansió $\quad 0 \to A \to B \qquad W_{0A} = 1/2\ s^E T^2$ $[J/m^2]$ $\qquad W_{AB} = -\frac{1}{2} s^D T^2$

T10 - RESONADORS i TRANSDUCTORS PIEZOELÈCTRICS

- Oscil·lació d'una barra

$$V_L = \sqrt{\frac{1}{\rho\, s^E}}$$

$$f_r = \frac{V_L}{2L} \qquad K = \frac{d}{\sqrt{s^E \varepsilon^T}} \qquad K^2 \approx \left(\frac{\pi}{2}\right)^2 \frac{f_a - f_r}{f_r}$$

freqüència resonància

freqüencia d'anti-ressonància

T11

$$\frac{\Delta f}{f_0} = \frac{2}{N-1}$$

$$L = \frac{\lambda}{2}$$