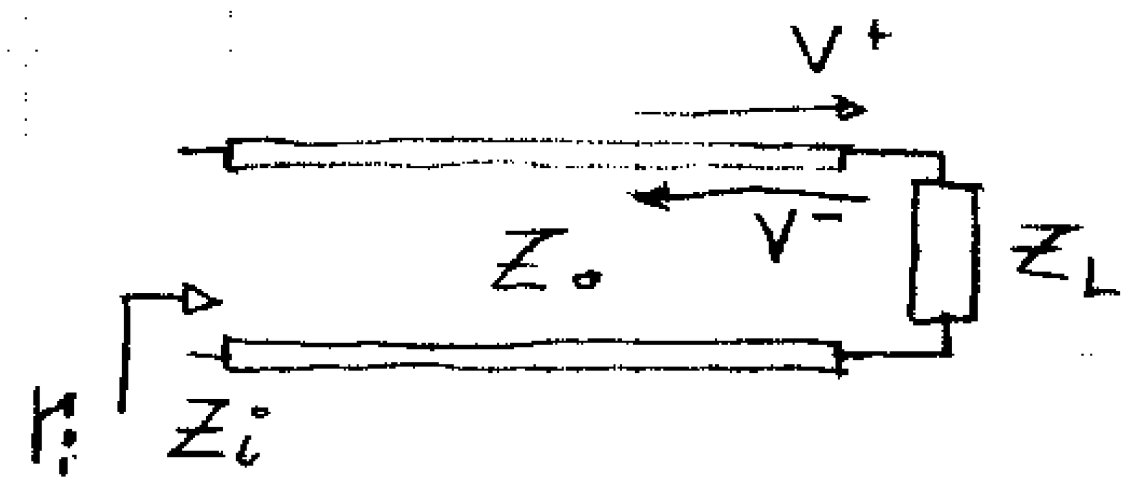
# MICROONDAS

## INTRODUCCIÓN



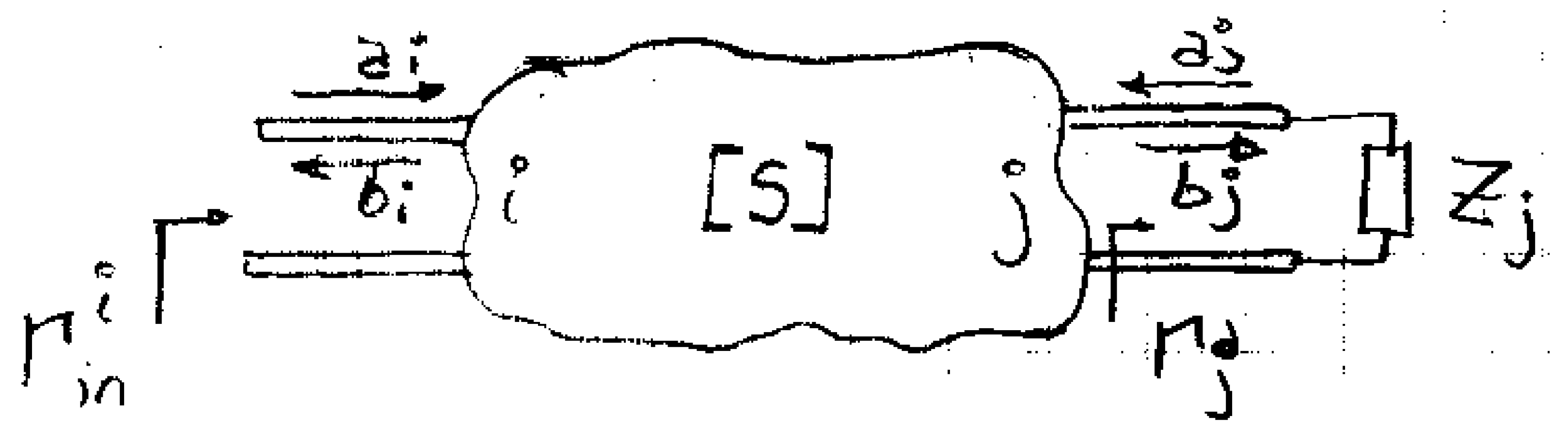

$$\boxed{\begin{aligned} V(z) &= V^+ e^{-j\beta z} + V^- e^{j\beta z} \\ I(z) &= \frac{1}{Z_o}\left(V^+ e^{-j\beta z} - V^- e^{j\beta z}\right) \end{aligned}}$$

$$\beta = \frac{\omega}{v_p} \quad ; \quad v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad ; \quad Z_o = \sqrt{\frac{L}{C}} = \frac{1}{v_p C}$$

$$\boxed{Z_i = Z_o \frac{Z_L + jZ_o \operatorname{tg}\beta\ell}{Z_o + jZ_L \operatorname{tg}\beta\ell}} \quad \boxed{\Gamma_i = \Gamma_L e^{-j2\beta\ell}} \quad \boxed{\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}}$$

$$\boxed{P = \frac{1}{2}\operatorname{Re}\{V \cdot I^*\} = \frac{1}{2}|I|^2 R} \quad \boxed{P_{av} = \frac{V_g^2}{8R_g}}$$

## PARÁMETROS S




$$\boxed{a_i = \frac{V_{oi}^+}{\sqrt{Z_{oi}}} e^{-j\beta z} \qquad b_i = \frac{V_{oi}^-}{\sqrt{Z_{oi}}} e^{+j\beta z}}$$

$$\boxed{V_i = \sqrt{Z_{oi}}(a_i + b_i) \qquad I_i = \frac{1}{\sqrt{Z_{oi}}}(a_i - b_i)}$$

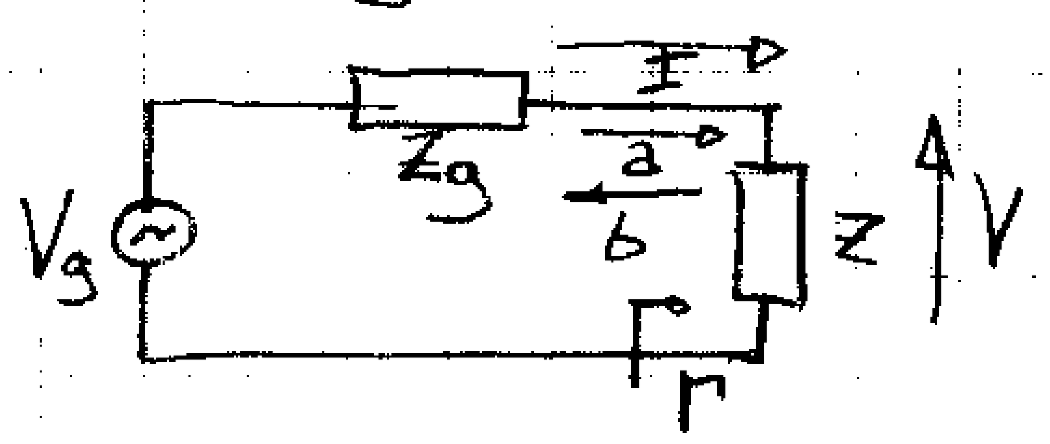
$$\boxed{\Gamma_{in}^i = \frac{b_i}{a_i} = \frac{Z_{in}^i - Z_{oi}}{Z_{in}^i + Z_{oi}}}$$ Coef. reflex a la entrada

$$\boxed{\Gamma_j = \frac{a_j}{b_j} = \frac{Z_j - Z_{oj}}{Z_j + Z_{oj}}}$$ Coef. reflex carga

$$\boxed{P_i = \frac{1}{2}\left(|a_i|^2 - |b_i|^2\right) = \frac{1}{2}|a_i|^2\left(1 - |\Gamma_{in}^i|^2\right)}$$ Pot. entregada pel generador a la port i

$$\boxed{P_j = \frac{1}{2}\left(|b_j|^2 - |a_j|^2\right) = \frac{1}{2}|b_j|^2\left(1 - |\Gamma_j|^2\right)}$$ Pot. dissipada per la càrrega en l'accés j

## Anàlisi generador




$$a = \frac{1}{2}\left(\frac{V}{\sqrt{Z_o}} + \sqrt{Z_o}\, I\right)$$

$$b = \frac{1}{2}\left(\frac{V}{\sqrt{Z_o}} - \sqrt{Z_o}\, I\right)$$

$$a = b_s \frac{1}{1 - \Gamma\Gamma_g} \qquad b = a\Gamma \qquad b_s = \frac{V_g}{\sqrt{Z_o}} \frac{Z_o}{Z_g + Z_o}$$

$$P_{av} = \frac{1}{2}|b_s|^2 \frac{1}{1 - |\Gamma_g|^2}$$

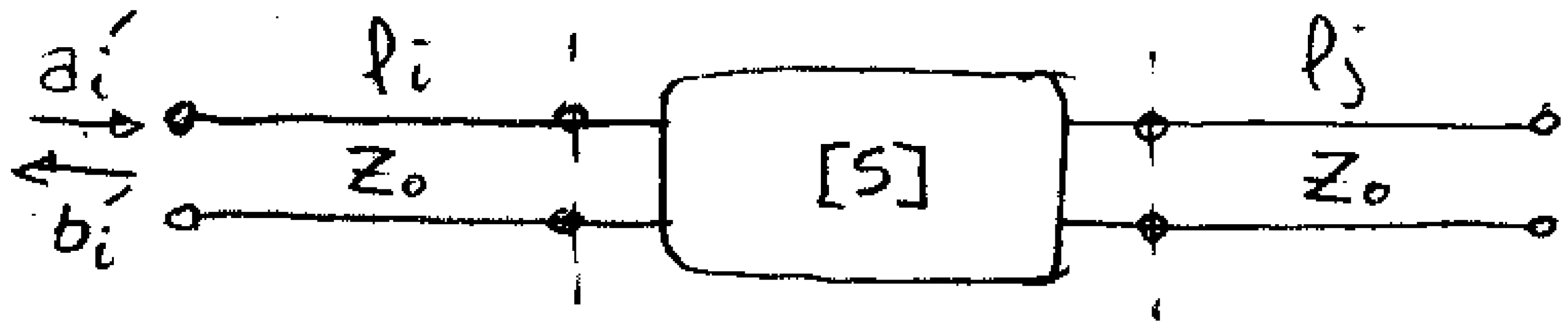
## - Matriu de paràmetres [S]

$$\boxed{S_{ij} = \frac{b_i}{a_j}\bigg|_{\substack{a_k=0 \\ k\neq j}}}$$

$a_k = 0$ equival a connectar $Z_o$ a la porta 'K'

$$\boxed{S_{ji} = \frac{V_j}{V_i}\bigg|_{\substack{a_k=0 \\ k\neq i}} (1 + S_{ii})}$$

## - Canvi de pla de referència



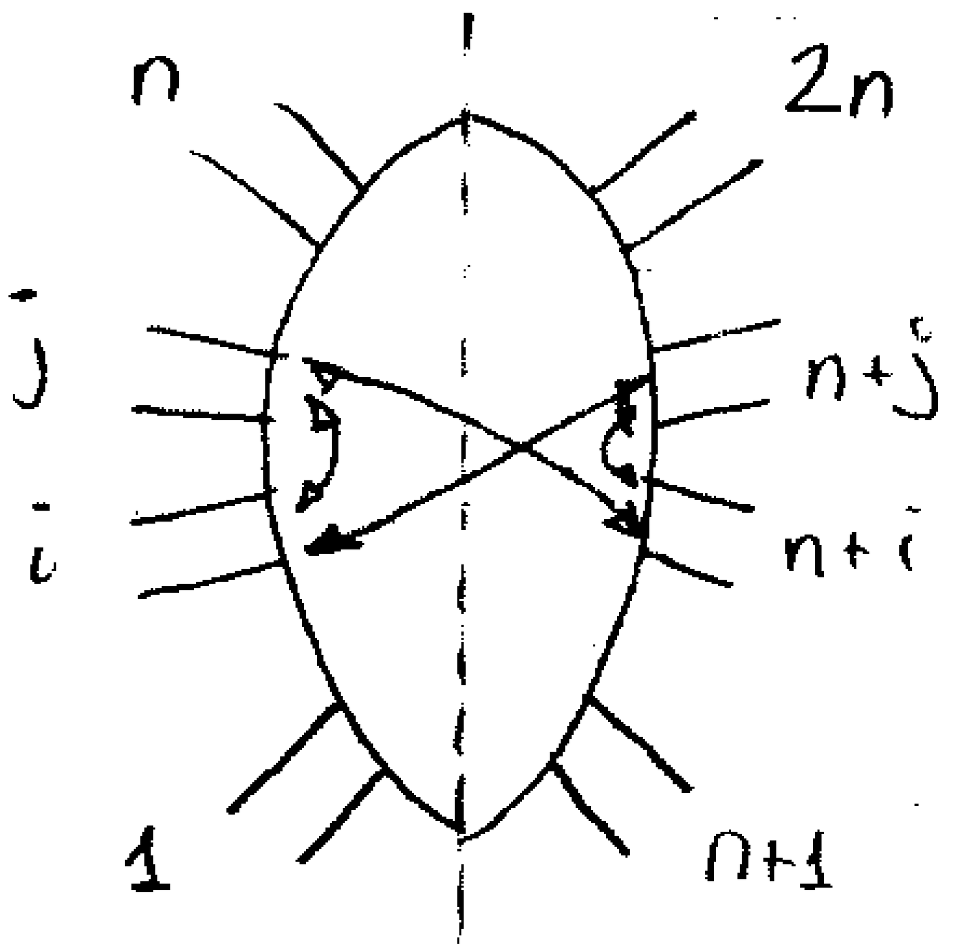

$$\boxed{S'_{ij} = S_{ij}\, e^{-j\beta(l_i + l_j)}}$$

## - Paràmetres S de circuits passius

Xarxa passiva $\Rightarrow P_d \geq 0 \quad \Rightarrow |S_{ij}| \leq 1$

Xarxa sense perdues $\Rightarrow P_d = 0 \quad \Rightarrow \boxed{[S][S]^H = Id}$

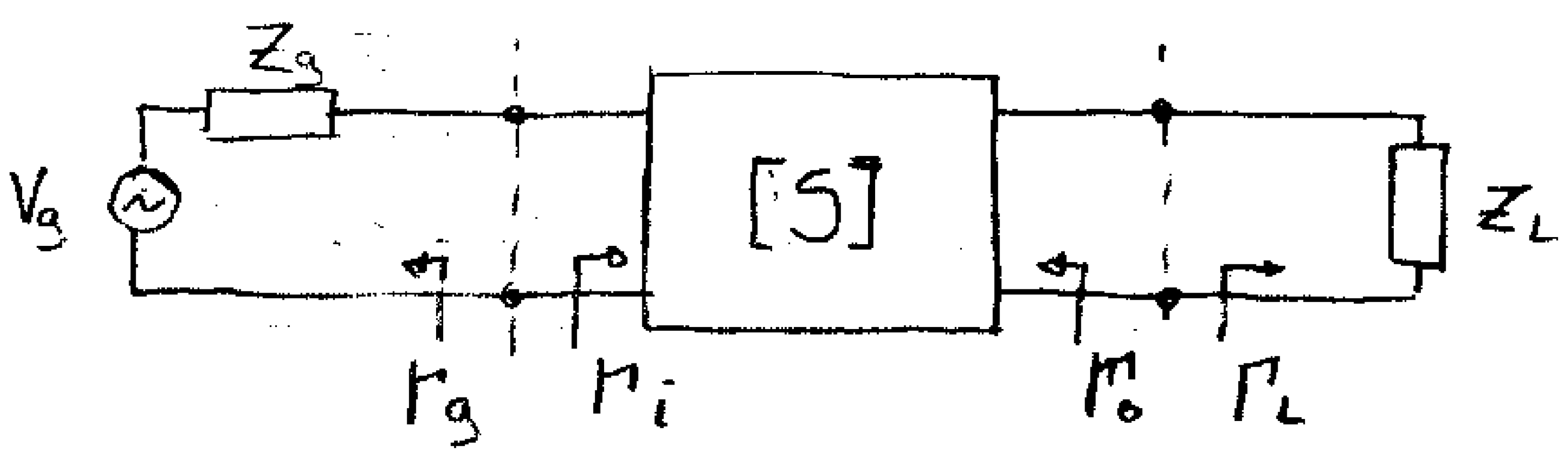
## - Paràmetres S de xarxes simètriques




$$\boxed{\begin{aligned} S_{ij} &= S_{n+i,\,n+j} \\ S_{i,\,n+j} &= S_{n+i,\,j} \end{aligned}}$$

Recíproques $S_{12} = S_{21}$

## ANÀLISI DE BIPORTS




$$\boxed{\Gamma_i = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}} \qquad \boxed{\Gamma_o = S_{22} + \frac{S_{12} S_{21} \Gamma_g}{1 - S_{11} \Gamma_g}}$$

## - Inversor d'impedàncies

$$\boxed{Z_i = \frac{\bar{K}^2}{Z_L}} \qquad \boxed{Y_i = \frac{\bar{J}^2}{Y_L}}$$

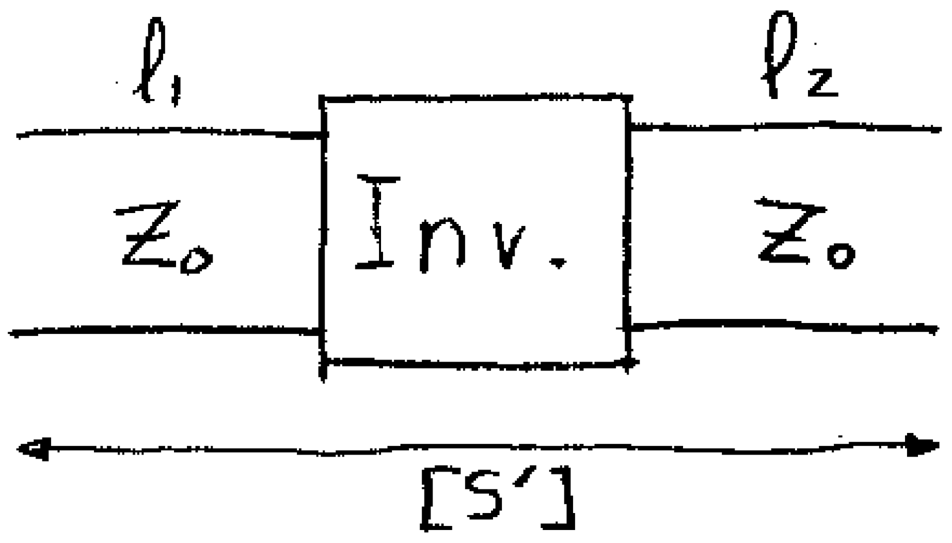
$\bar{K} < 1$ , $\bar{J} > 1 \Rightarrow$ Inv. impedàncies

$\bar{K} > 1$ , $\bar{J} < 1 \Rightarrow$ Inv. admitàncies

Xarxa recíproca i simètrica

$$[S] = \begin{bmatrix} \gamma & \pm j\sqrt{1-\gamma^2} \\ \pm j\sqrt{1-\gamma^2} & \gamma \end{bmatrix} \qquad \gamma = \frac{\bar{K}^2 - 1}{\bar{K}^2 + 1}$$

- Amb un inversor i dues línies es pot sintetitzar qualsevol xarxa sense perdues.




$$S_{11}' = \gamma \, e^{-j2\beta l_1}$$
$$S_{22}' = \gamma \, e^{-j2\beta l_2}$$

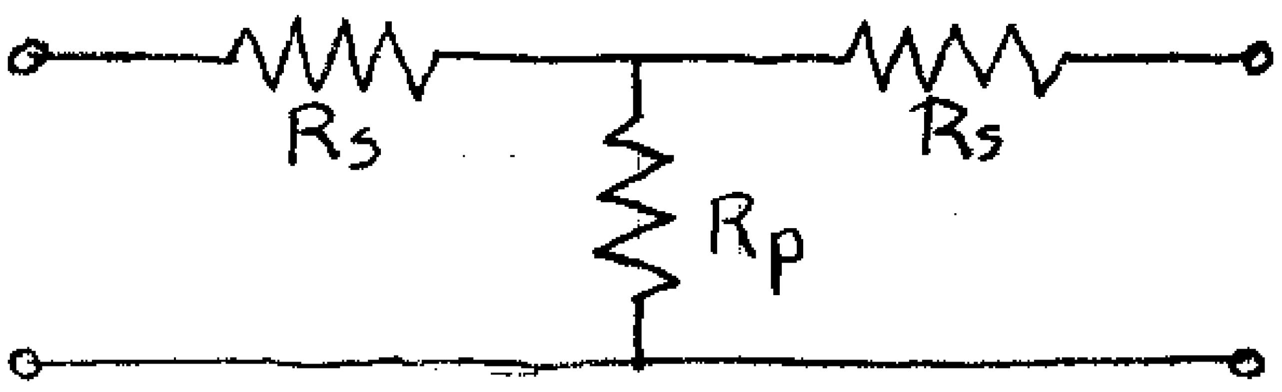
## - Atenuadors

Xarxa amb pèrdues, recíproca, completament adaptada ($S_{ii} = 0$)

$$[S] = \begin{bmatrix} 0 & S_{12} \\ S_{21} & 0 \end{bmatrix} \qquad P_L = P_{av} \, |S_{12}|^2 \qquad \text{Aten} = -10 \log |S_{21}|^2$$

S'utilitza per adaptar $\Rightarrow \Gamma_i = S_{21}^2 \, \Gamma_L$

- Disseny atenuador en T




$$R_p = \frac{Z_0^2 - R_s^2}{2R_s}$$

$$S_{21} = \frac{Z_0 - R_s}{Z_0 + R_s}$$

# XARXES DE 3 PORTES

## - DIVISORS DE POTÈNCIA

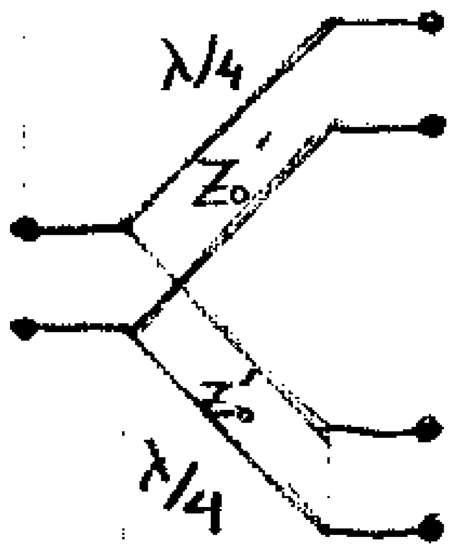
$$\boxed{\begin{aligned} P_2 &= P_{av}\,|S_{21}|^2 \\ P_3 &= P_{av}\,|S_{31}|^2 \end{aligned}}$$

Xarxes recíproques ($S_{ij} = S_{ji}$)

$$P_{av} = \frac{1}{2}|a_1|^2$$

### • Divisor de potència amb adaptadors $\lambda/4$

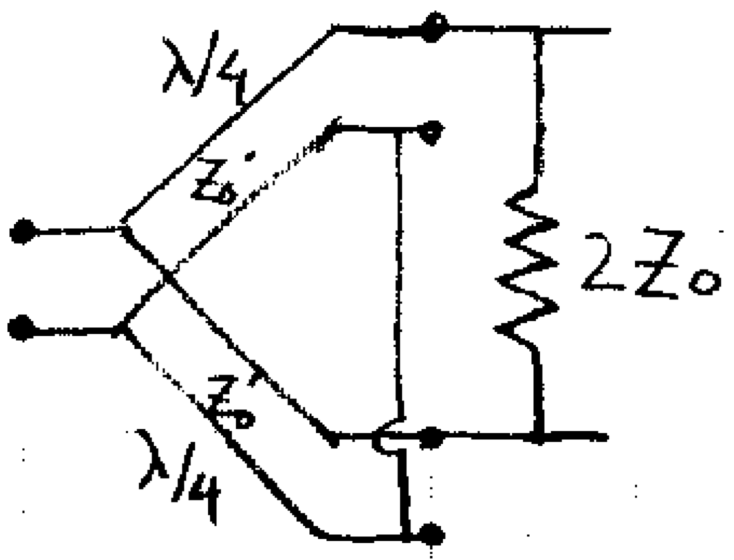
$$\boxed{Z_0' = \sqrt{2}\,Z_0}$$




$$\boxed{[S] = \begin{bmatrix} 0 & -j\,1/\sqrt{2} & -j\,1/\sqrt{2} \\ -j\,1/\sqrt{2} & 1/2 & -1/2 \\ -j\,1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}}$$

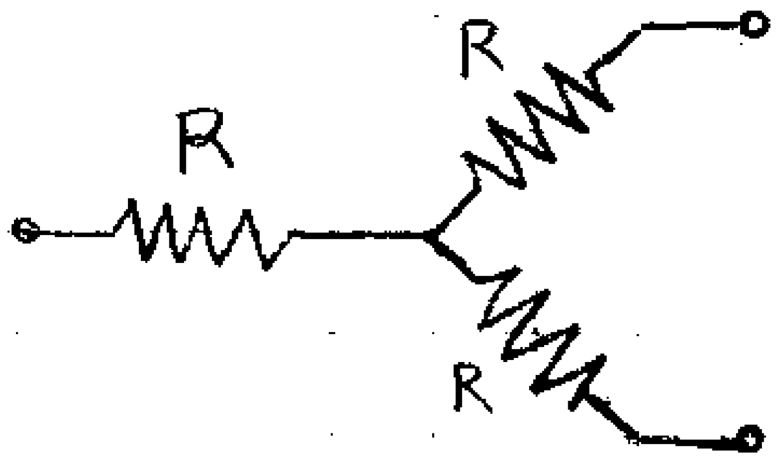
Divisor 3dB

### ◦ Divisor de Wilkinson (Desacobla portes sortida)




$$\boxed{[S] = \begin{bmatrix} 0 & -j\,1/\sqrt{2} & -j\,1/\sqrt{2} \\ -j\,1/\sqrt{2} & 0 & 0 \\ -j\,1/\sqrt{2} & 0 & 0 \end{bmatrix}}$$

### ◦ Divisor ~~r~~esistivo




Divisor 6dB

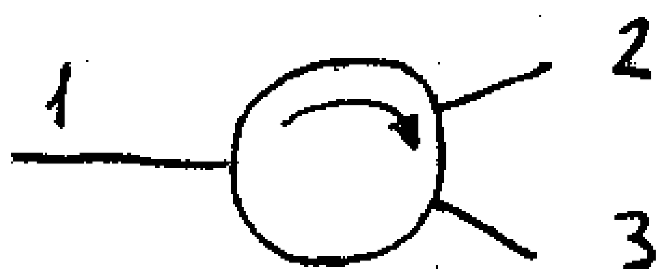
$$\boxed{[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}}$$

$$\boxed{R = \frac{Z_0}{3}}$$

## - CIRCULADORS

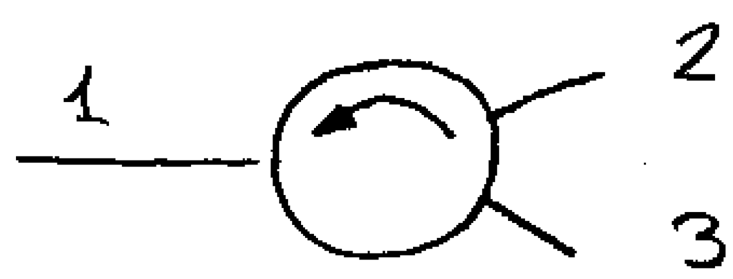
Xarxa sense perdues, NO recíproca, complétament adaptada ($S_{ii} = 0$)

$$[S] = \begin{bmatrix} 0 & 0 & e^{j\phi_1} \\ e^{j\phi_2} & 0 & 0 \\ 0 & e^{j\phi_3} & 0 \end{bmatrix}$$




ó

$$[S] = \begin{bmatrix} 0 & e^{j\phi_2} & 0 \\ 0 & 0 & e^{j\phi_3} \\ e^{j\phi_1} & 0 & 0 \end{bmatrix}$$




Aïllament $\quad I = -10\log|S_{12}|^2 \quad S_{23},\ S_{31}$

Perdues per retorn $\quad RL = -10\log|S_{ii}|^2 \quad -$

Perdues d'inserció $\quad IL = -10\log|S_{13}|^2 \quad S_{21},\ S_{32}$

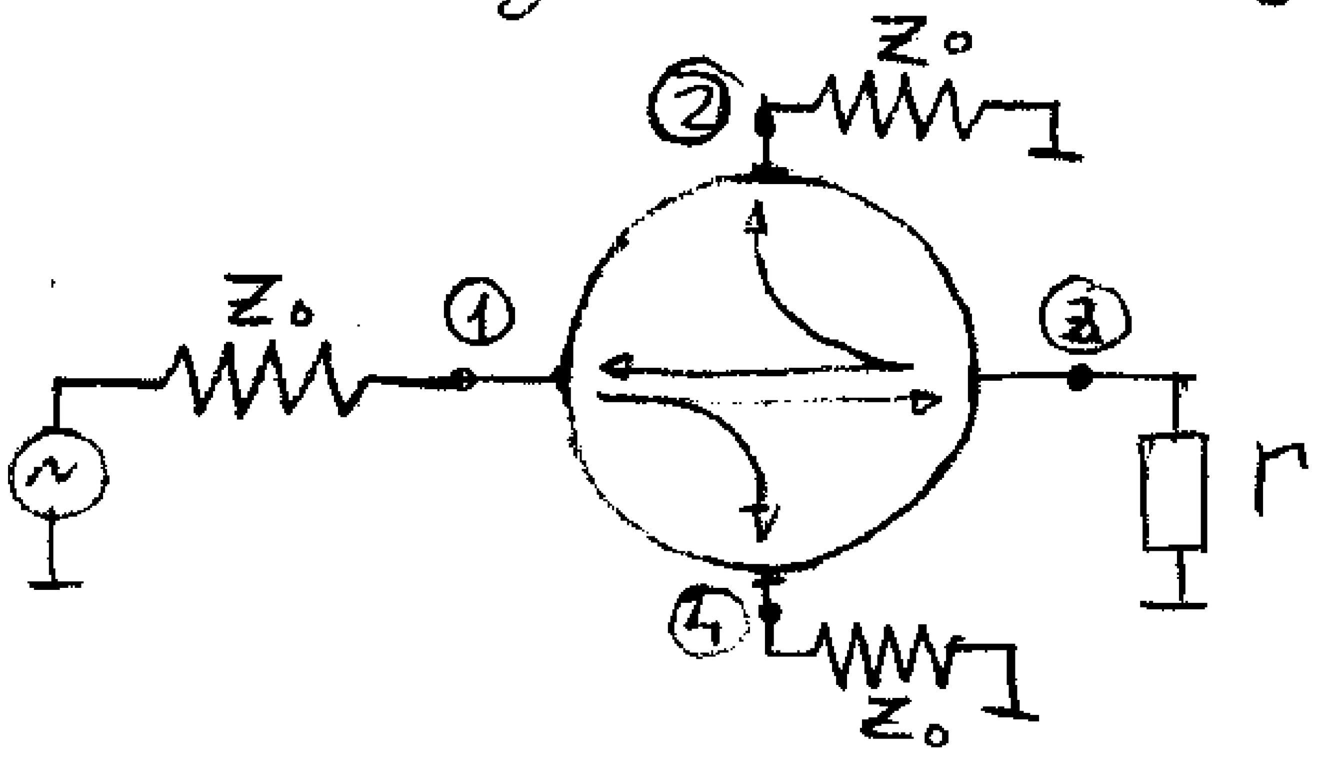
# XARXES DE 4 PORTES

- Xarxes sense perdues ($[S][S]^H = Id$),
  completament adaptades ($S_{ii} = 0$)
  recíproques ($S_{ij} = S_{ji}$)

$$[S] = \begin{bmatrix} 0 & 0 & \tau e^{j\theta_1} & k e^{j\varphi_1} \\ 0 & 0 & K e^{j\varphi_2} & \tau e^{j\theta_2} \\ \tau e^{j\theta_1} & K e^{j\varphi_2} & 0 & 0 \\ K e^{j\varphi_1} & \tau e^{j\theta_2} & 0 & 0 \end{bmatrix} \qquad \begin{array}{l} K^2 + \tau^2 = 1 \\ \theta_1 + \theta_2 = \varphi_1 + \varphi_2 \pm \pi \end{array}$$

- Acoblador direccional $K \ll 1 \ ; \ \tau \approx 1$

* Mesura de coeficients de reflexió




$$\Gamma_m \triangleq \frac{b_2}{b_4} = \tau e^{j(\varphi_2 - \varphi_1 + \theta_1)}\,\Gamma$$

$$\boxed{\Gamma_m = A\,\Gamma}$$

Calibració $\Gamma = \Gamma_c \Rightarrow A = \dfrac{\Gamma_{mc}}{\Gamma_c}$ $\qquad \boxed{\Gamma = \dfrac{\Gamma_c}{\Gamma_{mc}}\,\Gamma_m}$

* Disseny amb línies acoblades

$$C = \varepsilon_0 \varepsilon_r \frac{\text{Àrea}}{\text{Distància}}$$


Mode parell $\Rightarrow$ $Z_{oe} = \dfrac{1}{v_{pe}\, C_e}$ $\qquad C_e = C_a$

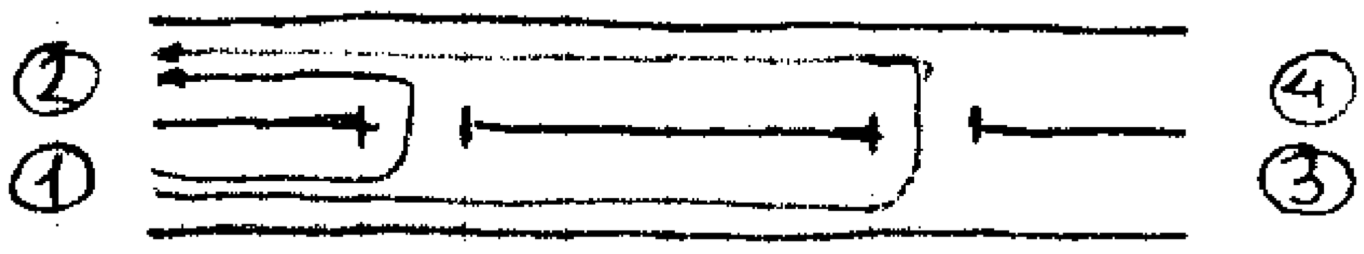
Mode imparell $\Rightarrow$ $Z_{oo} = \dfrac{1}{v_{po}\, C_o}$ $\qquad C_o = C_a + 2C_{ab}$

$$\boxed{\varepsilon_{re} = \varepsilon_{ro}} \qquad \boxed{Z_{oe}\, Z_{oo} = Z_0^2}$$

$$\tau = \boxed{S_{21} = \frac{\sqrt{1-\alpha^2}}{\sqrt{1-\alpha^2}\cos\beta l + j\sin\beta l}} \qquad K = \boxed{S_{31} = \frac{j\alpha \sin\beta l}{\sqrt{1-\alpha^2}\cos\beta l + j\sin\beta l}}$$

$$\boxed{\alpha = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}}$$

## *Disseny amb guies d'ona

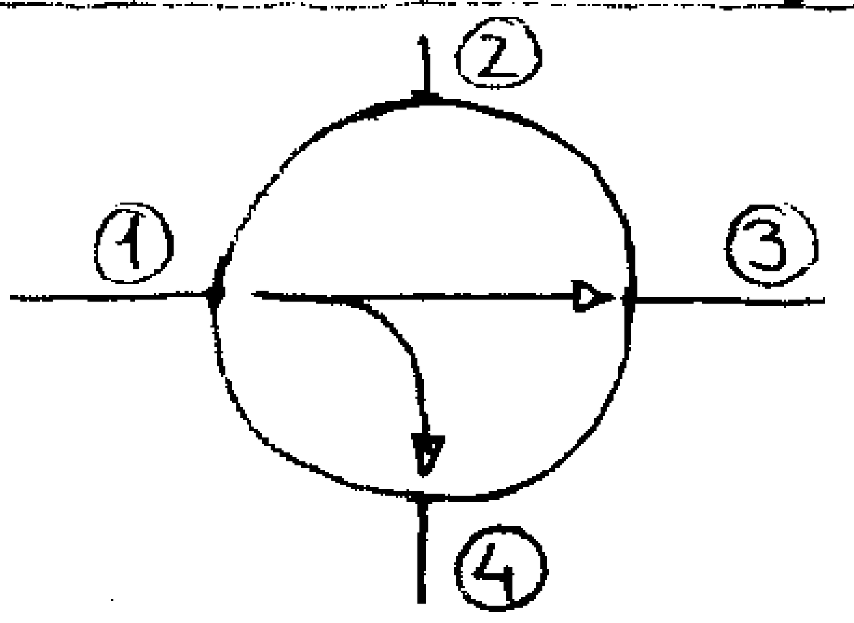



$$\lambda_c = 2a\sqrt{\varepsilon_r}$$

$$\boxed{\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}}$$

Acoblador direccional $\Leftrightarrow$ $\boxed{l = \lambda_g/4}$

## *Limitacions acobladors




Aïllament $\Rightarrow I = -10 \log |S_{21}|^2$

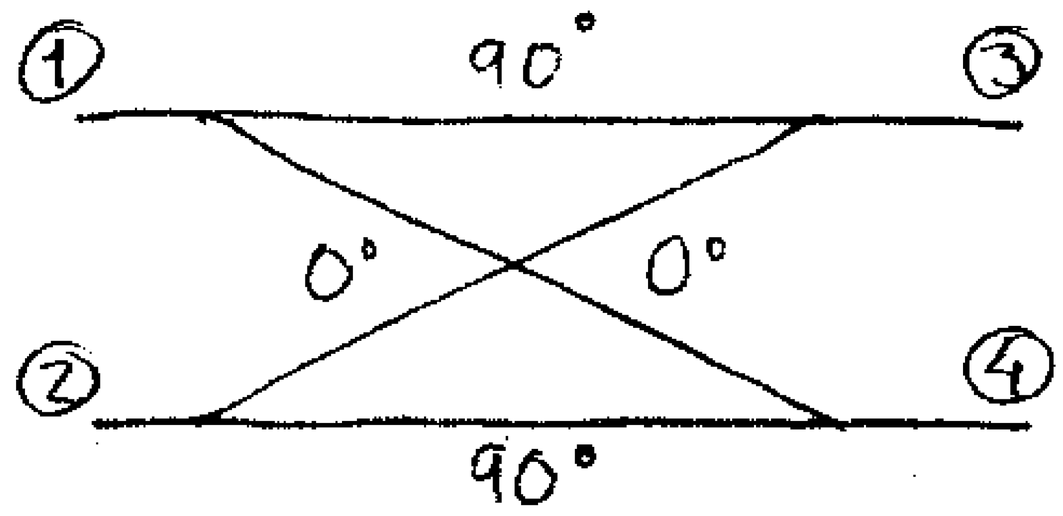
Acoblament $\Rightarrow C = -10 \log |S_{41}|^2$

Perdues retorn $\Rightarrow RL = -10 \log |S_{ii}|^2$

Directivitat $\Rightarrow D = I - C$

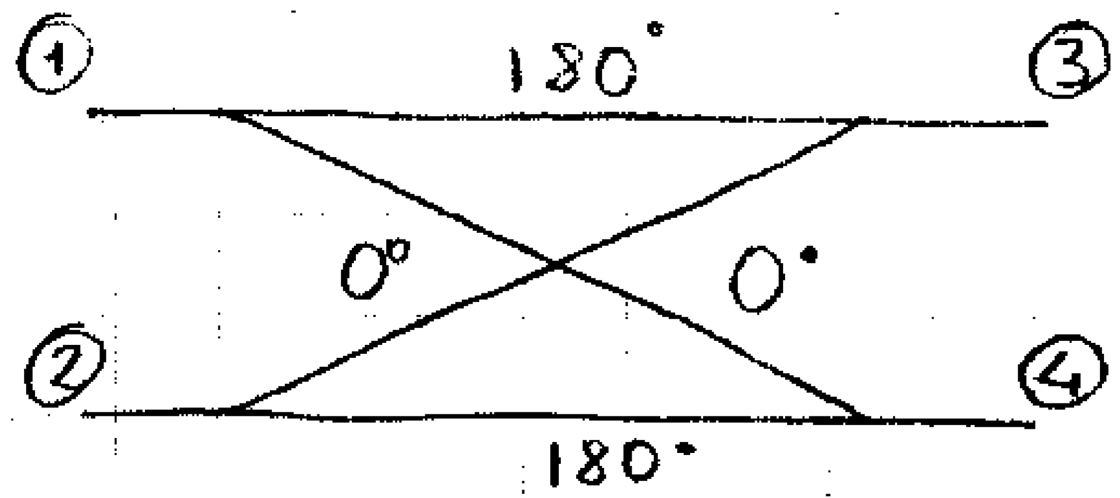
# - Hibrids $\quad K = \tau = 1/\sqrt{2} \quad (C = 3\,dB)$

## *Hibrid 90°



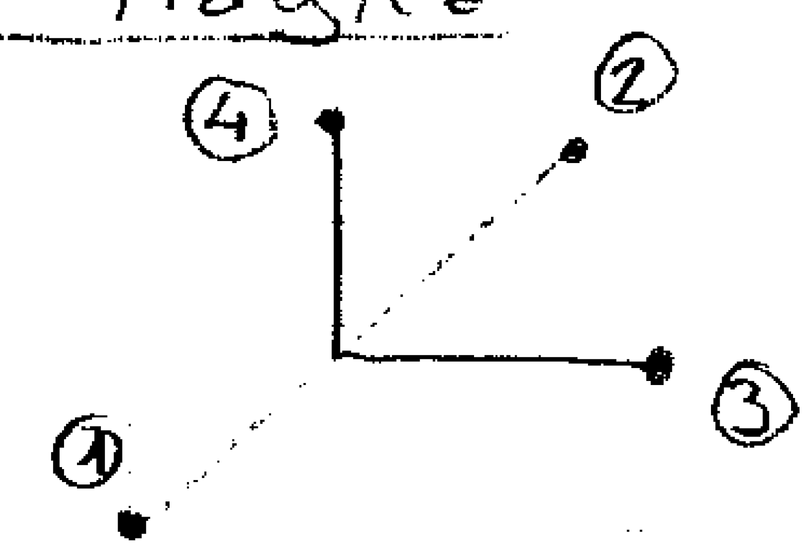

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & j & 1 \\ 0 & 0 & 1 & j \\ j & 1 & 0 & 0 \\ 1 & j & 0 & 0 \end{bmatrix}$$

## *Hibrid 180°



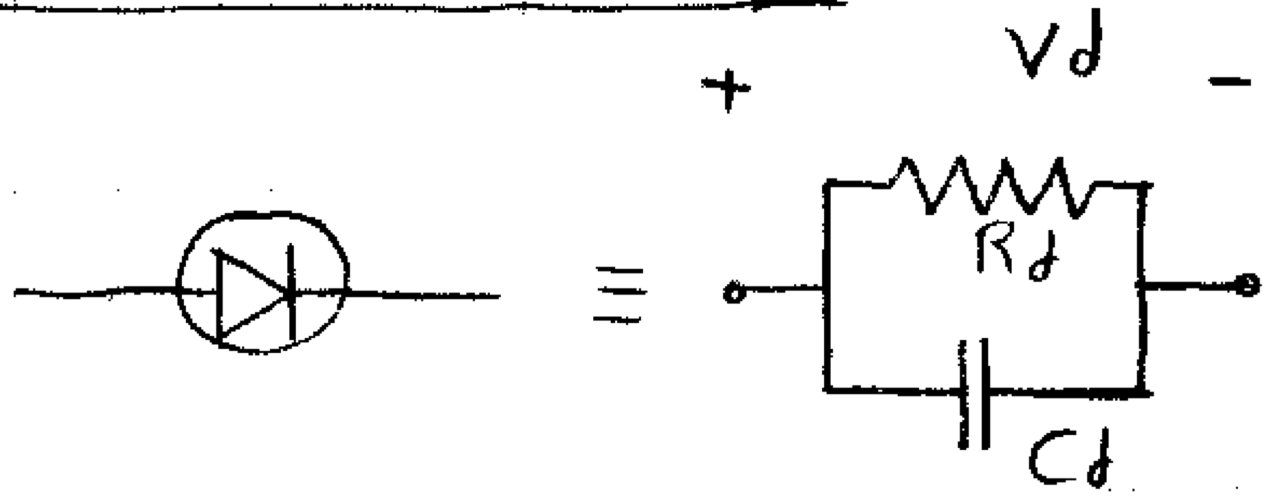

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 \\ +1 & -1 & 0 & 0 \end{bmatrix}$$

## * T-Màgica




$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

## - Diodes PIN

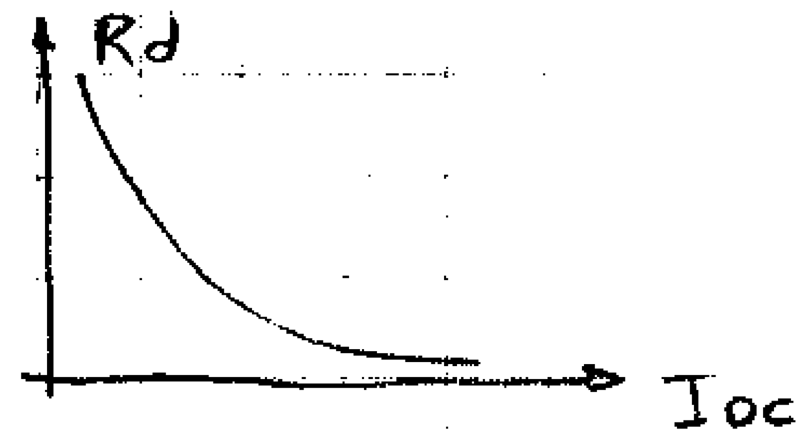



$v_d > 0$ Diode en directe : Comportament resistiu

$v_d < 0$ Diode en inversa : Comportament capacitiu

### - Diode en petit senyal (RPS)

$$\boxed{R_d(I_{DC}) = \frac{nKT}{q_e}\frac{1}{I_{DC}+I_s}}$$




$$\boxed{C_d(V_{DC}) = \frac{C_0}{\sqrt{1-\frac{V_{DC}}{\Phi}}}}$$

$I_s$ : Corrent sat invers
$\Phi$ : Potencial contacte $\simeq 0'7V$
$n$ : Factor idealitat

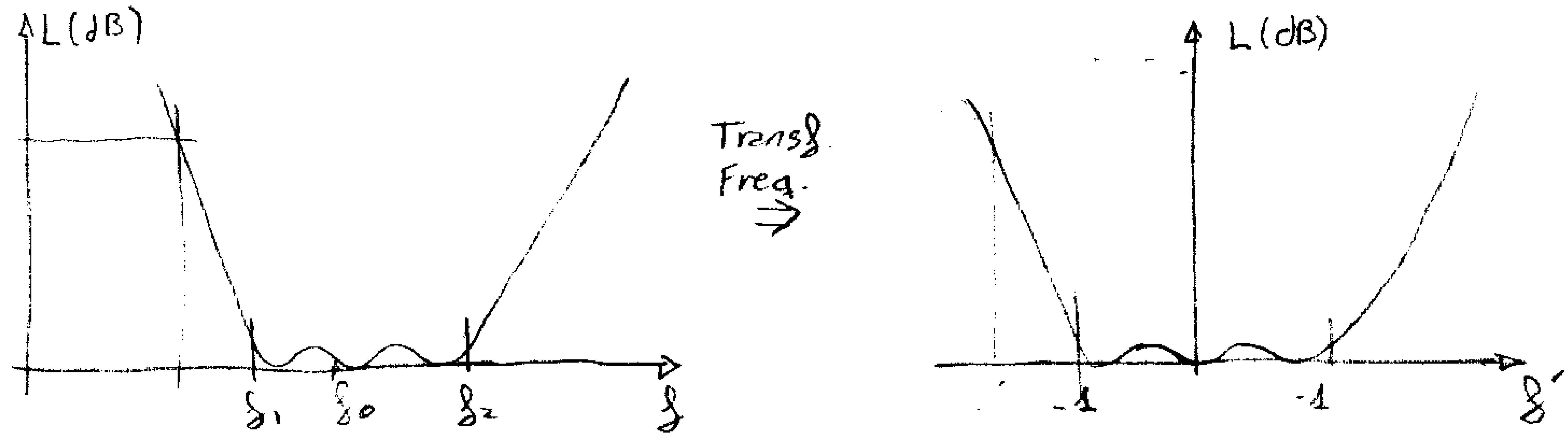
### - Diode en gran senyal

$$v_d = V_d\cos(\omega t)$$

Terme continua : $i_{DC} = \frac{1}{2}\left(\frac{q_e}{nKT}\right)^2 I_s V_d^2$

Terme a $\omega$ : $i_\omega = \frac{1}{R_d(0)} V_d\cos(\omega t)$

# FILTRES A FREQÜÈNCIES DE $\mu$O



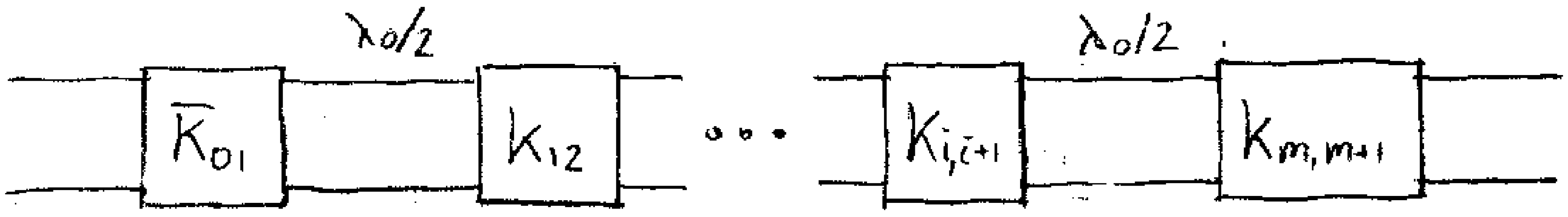

Transformació freqüencial:

$$\boxed{f_0 = \sqrt{f_1 \cdot f_2}} \qquad \boxed{W = \frac{f_2 - f_1}{f_0}} \qquad \boxed{\omega' = \frac{1}{W}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Transformació

$$\boxed{L(dB) = -10 \log |S_{21}|^2}$$

- Disseny amb inversors impedàncies/admitàncies



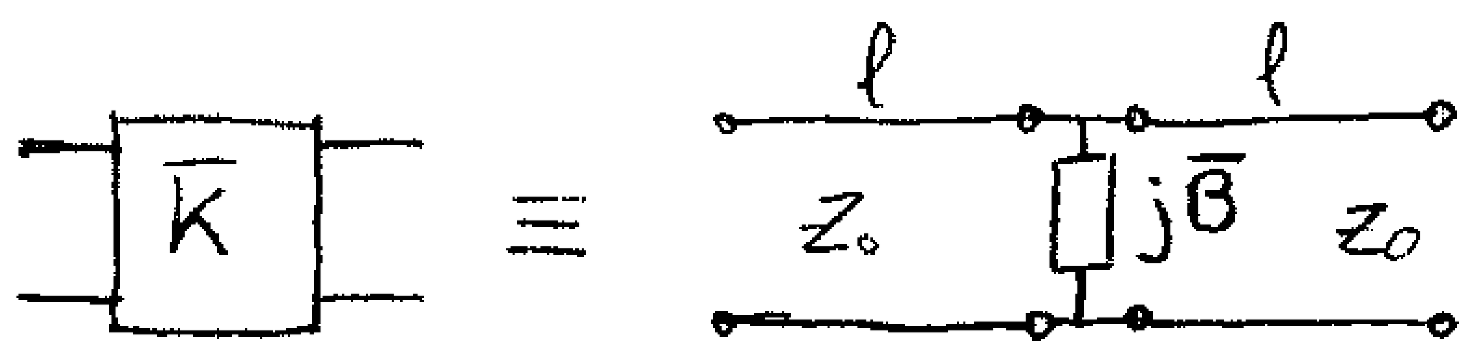

$$\boxed{\bar K_{01} = \sqrt{\frac{\pi W}{2 g_1}} \qquad \bar K_{i,i+1} = \frac{\pi W}{2\sqrt{g_i g_{i+1}}} \qquad \bar K_{m,m+1} = \sqrt{\frac{\pi W}{2 g_m g_{m+1}}}}$$

* Protoip ordre senar ⇒ $g_1 \quad g_2 \quad g_3 \quad \cdots \quad g_{n-2} \quad g_{n-1} \quad \underset{1}{g_n}$

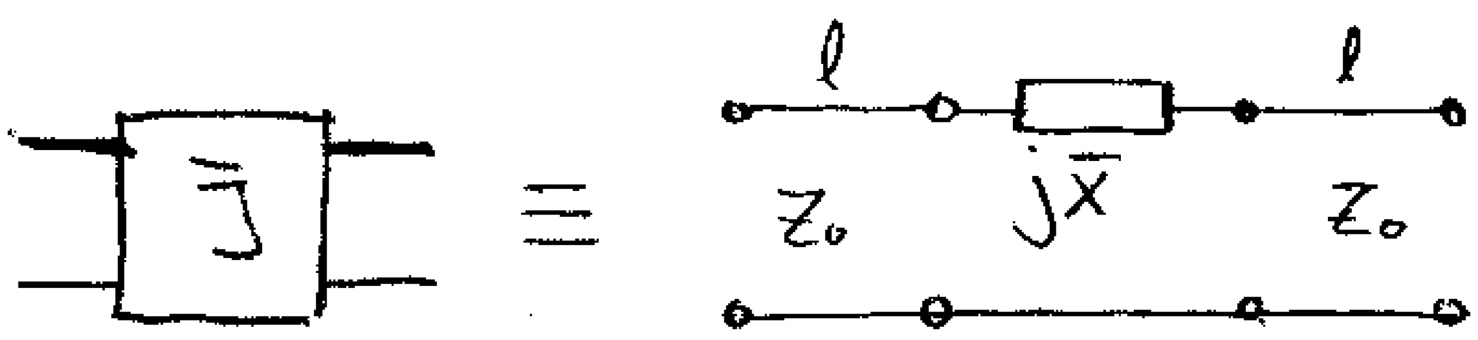
* Per inversors admitàncies canviem $\bar K$ per $\bar J$

- Disseny de inversors impedàncies/admitàncies

$\bar B = \frac{B}{Z_0}$

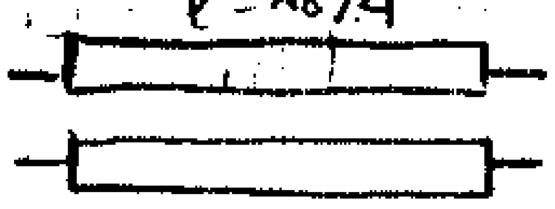



$$\boxed{|\bar B| = \frac{1 - \bar K^2}{\bar K}} \qquad \boxed{\frac{l}{\lambda} = \frac{1}{4\pi} \operatorname{arctg} \frac{2}{\bar B}}$$



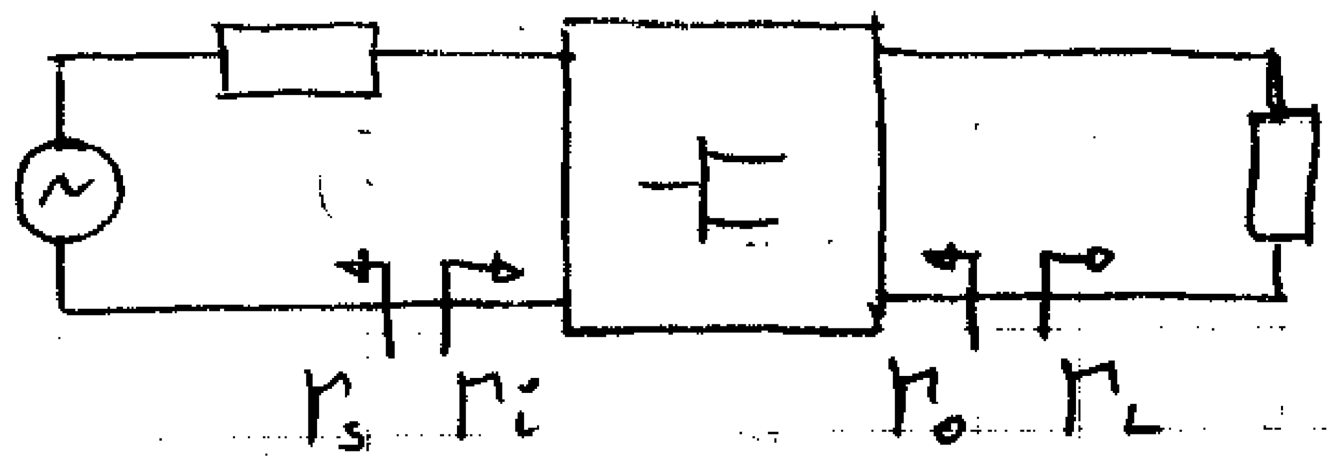

$$\boxed{|\bar X| = \frac{1 - \bar J^2}{\bar J}} \qquad \boxed{\frac{l}{\lambda} = \frac{1}{4\pi} \operatorname{arctg} \frac{2}{\bar X}}$$

- Inversor admitàncies amb línies acoblades




$$\boxed{\bar Z_{oe} = \bar J + \sqrt{1 + \bar J^2}} \qquad \boxed{\bar Z_{oo} = 1/\bar Z_{oe}}$$

# AMPLIFICADORS



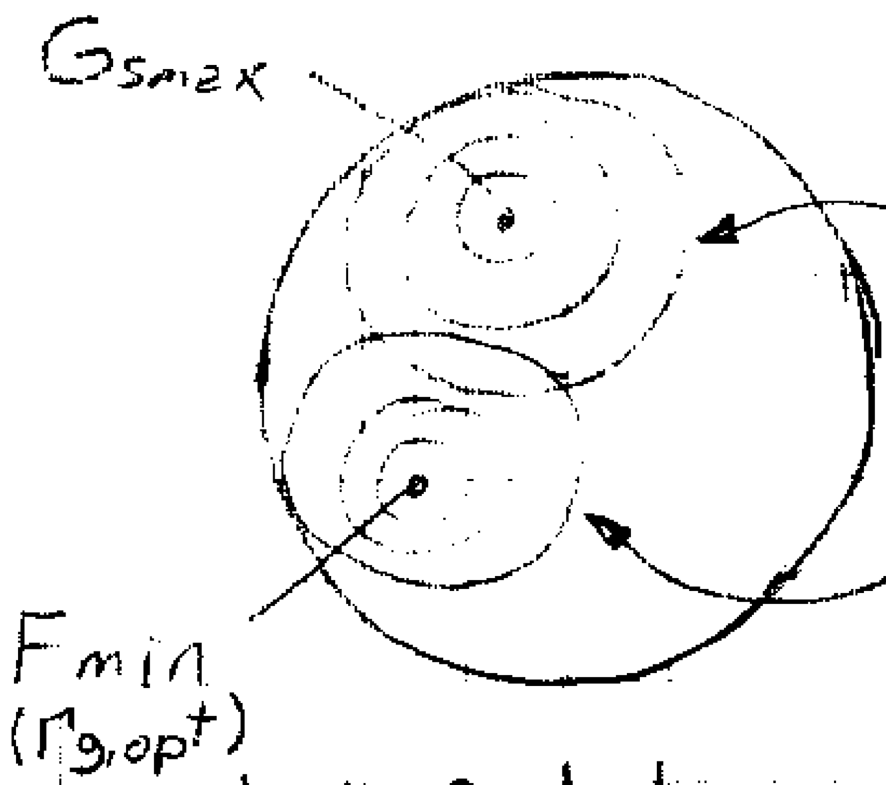

$$\boxed{G_T = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-S_{11}\Gamma_s)(1-S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_s\Gamma_L|^2}}$$

Guany màxim $\Rightarrow$ Adaptació conjugada

$$\boxed{\begin{aligned} \Gamma_g &= \Gamma_i^* = \left( S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} \right)^* \\ \Gamma_L &= \Gamma_o^* = \left( S_{22} + \frac{S_{12} S_{21} \Gamma_g}{1 - S_{11}\Gamma_g} \right)^* \end{aligned}}$$

Aproximació unilateral $(S_{12} = 0) \Rightarrow \Gamma_g = S_{11}^*$ ; $\Gamma_L = S_{22}^*$

- Cercles de guany cte, soroll cte

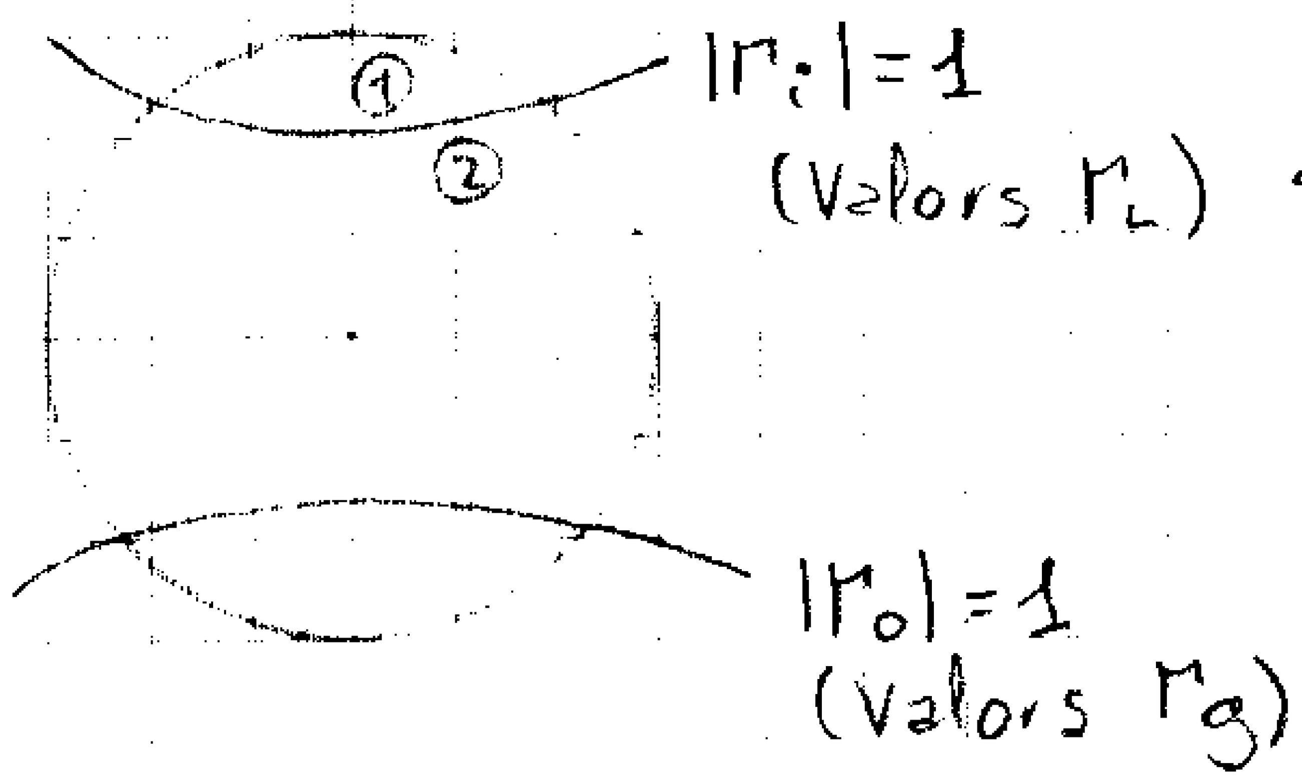



Compromis Guany $\leftrightarrow$ Soroll

Valor de $\Gamma_g$ que fan Gs cte

" " F "

- Estabilitat




① Si $|S_{11}| < 1 \Rightarrow |\Gamma_i| < 1$ Estable

② Si $|S_{11}| > 1 \Rightarrow |\Gamma_i| > 1$ Inestable

Si $|S_{22}| < 1 \Rightarrow |\Gamma_o| < 1$ Estable

◦ Factor d'estabilitat

$K > 1$ Estable
$K < 1$ Inestable

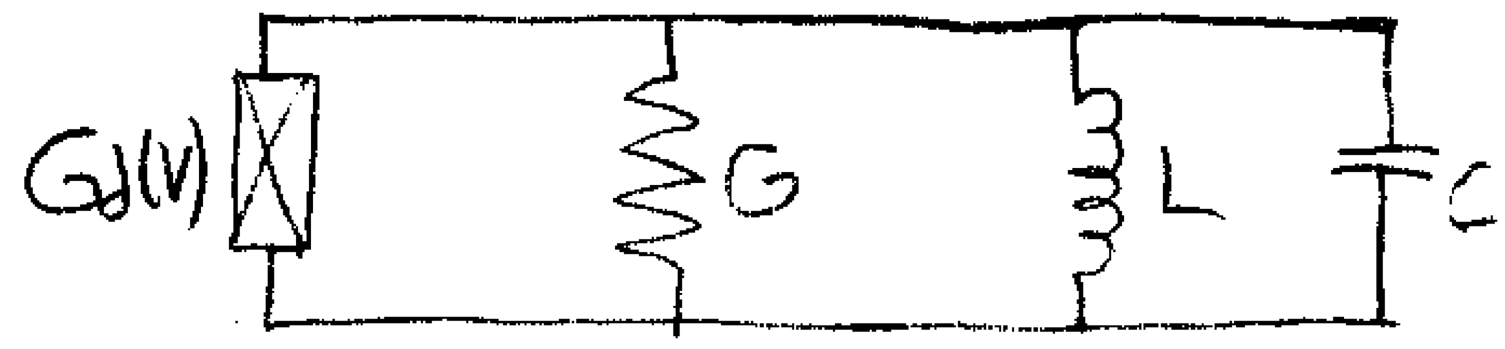
$$\boxed{G_{T\,max} = \left| \frac{S_{21}}{S_{12}} \right| \left( K - \sqrt{K^2 - 1} \right)}$$

$$\boxed{F = F_{min} + 4R_n \frac{|\Gamma_s - \Gamma_{opt}|^2}{(1-|\Gamma_s|^2)|1+\Gamma_{opt}|^2}}$$

(Formula de la K bastante complicada)

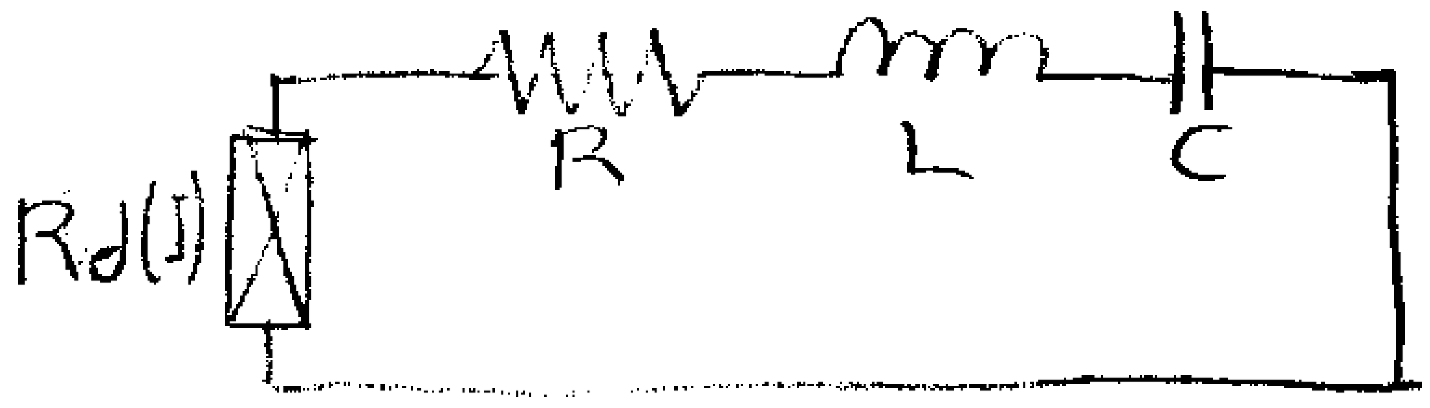
# OSCIL·LADORS

## - Mètode de disseny de la resistència negativa




$$G_d = -R \Rightarrow \text{Oscil·lador}$$

$$\boxed{G_d(V) = G_{d_0} + bV^2}$$




$$f_{osc} = \frac{1}{2\pi\sqrt{LC}}$$

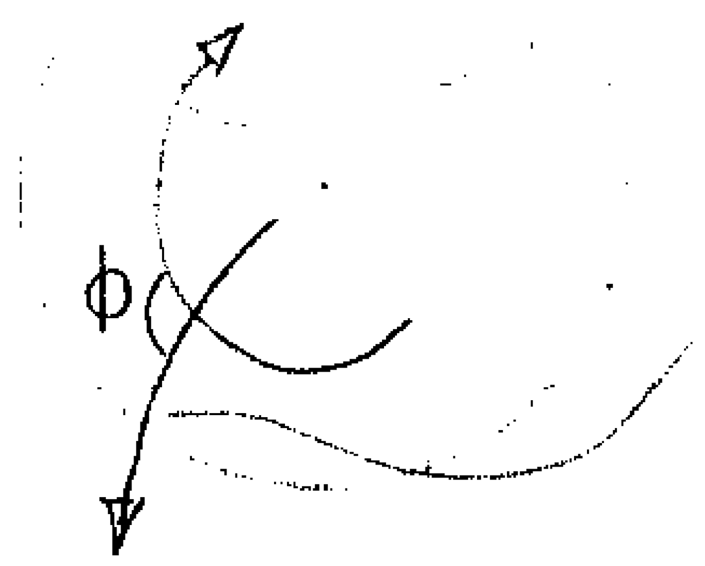
$$Q_{serie} = \frac{\omega_0 L}{R} \qquad Q_{paral·lel} = \frac{\omega_0 C}{G}$$

### • Potència oscil·lació

$$P_{max} \Leftrightarrow \boxed{G_{opt} = -\frac{G_{d_0}}{2}} \qquad \boxed{P_{max} = \frac{G_{d_0}^2}{8b}}$$

## - Línia de dispositiu i línia de càrrega

línia càrrega




línia dispositiu $\quad P = \frac{1}{2} V^2 G = -\frac{1}{2} V^2 (G_{d_0} + bV^2)$

$$\Phi_{opt} = 90^\circ$$