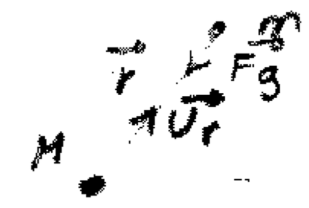
MECÁNICA

FUERZAS

Gravitatoria $\vec{F_g} = -G\frac{mM}{r^2}\vec{u}$



Electromagnética $\vec{F} = K_e\frac{Qq}{r^2}\vec{u_r} = q\vec{E}$

LEYES DE NEWTON

1ª → Si $\vec{F}=0 \Rightarrow \vec{v}=cte$ Para partículas libres (sin Fuerzas) $\Rightarrow \vec{a}=0$

2ª → $\boxed{\sum \vec{F}_{ext} = \frac{d\vec{p}}{dt}} = \boxed{m\cdot\vec{a}}$

3ª → $\boxed{\vec{F}_{12} = -\vec{F}_{21}}$ $\boxed{\vec{p}_{tot} = cte}$

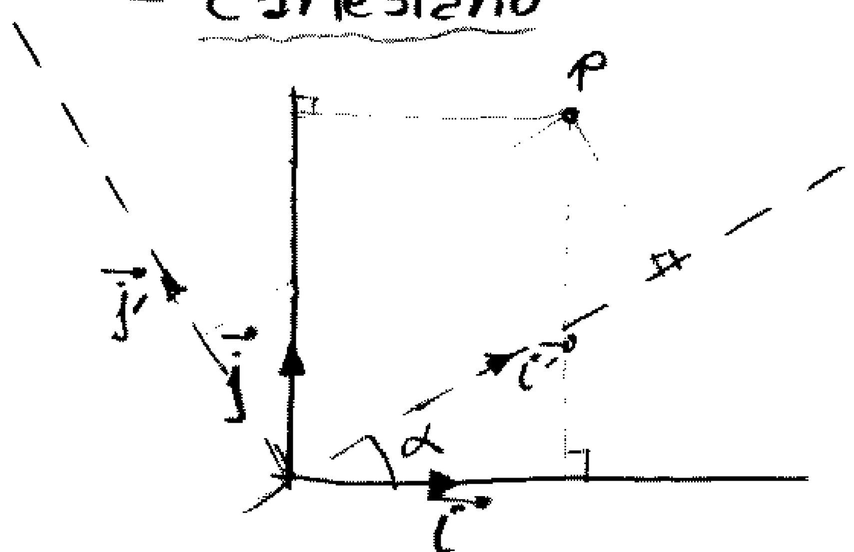
SISTEMAS DE REFERENCIA INERCIALES

$\boxed{\vec{a}=0}$ "de un S.Ref sobre el otro"

- Cambio Origen coordenadas "Con mov. uniforme de S'/S" $\boxed{\vec{p}=cte}$
- Giro $\boxed{\vec{L}=cte}$ $\boxed{\forall t \to E=cte}$
- Rotación $\forall \vec{v}=cte$

SISTEMAS DE COORDENADAS

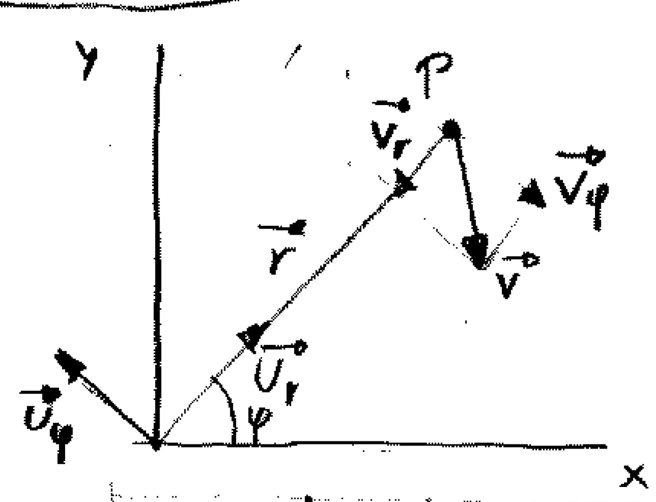
- Cartesiano



$$\begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}\begin{pmatrix} \vec{i} \\ \vec{j} \end{pmatrix} = \begin{pmatrix} \vec{i}' \\ \vec{j}' \end{pmatrix}$$

$(x,y) \to \vec{r} = x\vec{i} + y\vec{j}$

- Polares

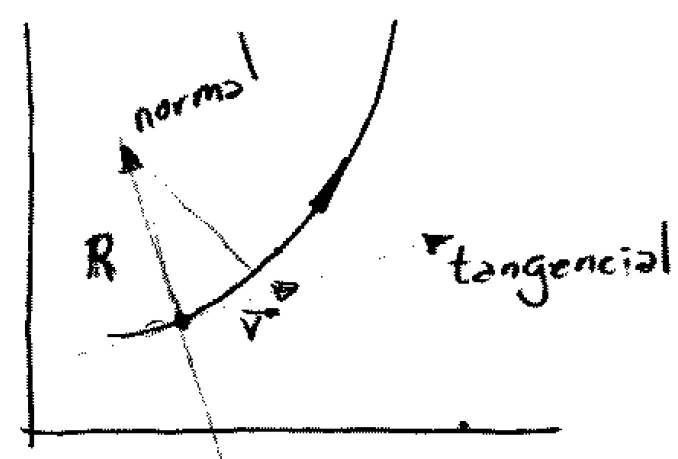


$V_r = \frac{dr}{dt}$ $V_\varphi = r\omega$

$\boxed{V = V_r\vec{U_r} + V_\varphi\vec{U_\varphi}}$ $\boxed{U_r = \frac{\vec{r}}{|r|}}$

$(r,\varphi) \to \vec{r} = r\vec{U_r}$

- Intrínsecas



$\vec{a}_t = \frac{d\vec{v}}{dt}\vec{U_t}$ $a_n = \frac{v^2}{R}\vec{U_n}$ $\boxed{U_t = \frac{\vec{v}}{|\vec{v}|}}$

$\vec{V} = v\vec{U_t}$ $v = |\vec{v}| \to$ celeridad

TRABAJO Y ENERGÍA CINÉTICA

$\boxed{W = \frac{1}{2}m\Delta v^2} = E_{kf} - E_{k0}$ $\boxed{P = \vec{F}\cdot\vec{v} = \frac{dW}{dt}}$ $\boxed{W = \int F\cdot d\vec{r}}$

FUERZAS CONSERVATIVAS Y ENERGÍA POTENCIAL

$\boxed{\Delta U = -\int_A^B F\,d\vec{r}} = -W_{cons}$ $\boxed{F = -\nabla U = -\left(\frac{\partial U}{\partial x}i + \frac{\partial U}{\partial y}j + \frac{\partial U}{\partial z}k\right)}$ $\boxed{\Delta U = -F\Delta r\cos\alpha}$

$$\boxed{\vec{F} = -\left(\frac{\partial U}{\partial r}\vec{U_r} + \frac{\partial U}{R\partial\theta}\vec{U_\varphi}\right)}$$

MOMENTO CINÉTICO

F puramente radial $\Rightarrow \boxed{\vec{L}=cte}$

- Momento de una Fuerza

$\boxed{\vec{M} = \vec{r}\wedge\vec{F}} = rF\sin\theta k = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

$\boxed{\vec{L} = \vec{r}\wedge\vec{p}}$ $\boxed{\vec{M} = \frac{d\vec{L}}{dt}}$

- En un M.C.U

$\boxed{\vec{L} = -m\omega r^2 \Rightarrow cte \Rightarrow \vec{M}=0}$

Sistemas Partículas

$\boxed{\vec{r}_{CM} = \frac{\sum \vec{r}_i m_i}{M}}$; $\vec{V}_{CM} = \frac{\vec{P}}{M} \Rightarrow \boxed{\vec{p} = M\vec{V}_{CM}}$; $\frac{d\vec{p}}{dt} = \vec{F}_{ext}$

$\Delta(E_K + U_{ext} + \text{Autoenergía}) = \boxed{\Delta E = W_{no.cons.}}$

- Cambio s. ref

$SR.CM \rightarrow \boxed{E_K = \overset{SR.LAB}{E_K'} + \frac{1}{2} M v_{CM}^2}$

Termodinámica

$\boxed{PV = nRT}$

$R = kN_A = 8'31\ J/K\,mol$; $1eV = 1'6 \cdot 10^{-19}\ J$

$KT(T=300K) \approx 26 meV$; $E = qV = e^- \cdot 1V$

$\boxed{p = \frac{F}{S}}$

$n = \frac{N}{N_A}$; $N_A = 6'023 \cdot 10^{23}\ \text{moleculas}/mol$ — $n \rightarrow$ nº moles; $N \rightarrow$ part. totales

$K = 1'38 \cdot 10^{-23}\ J/K$

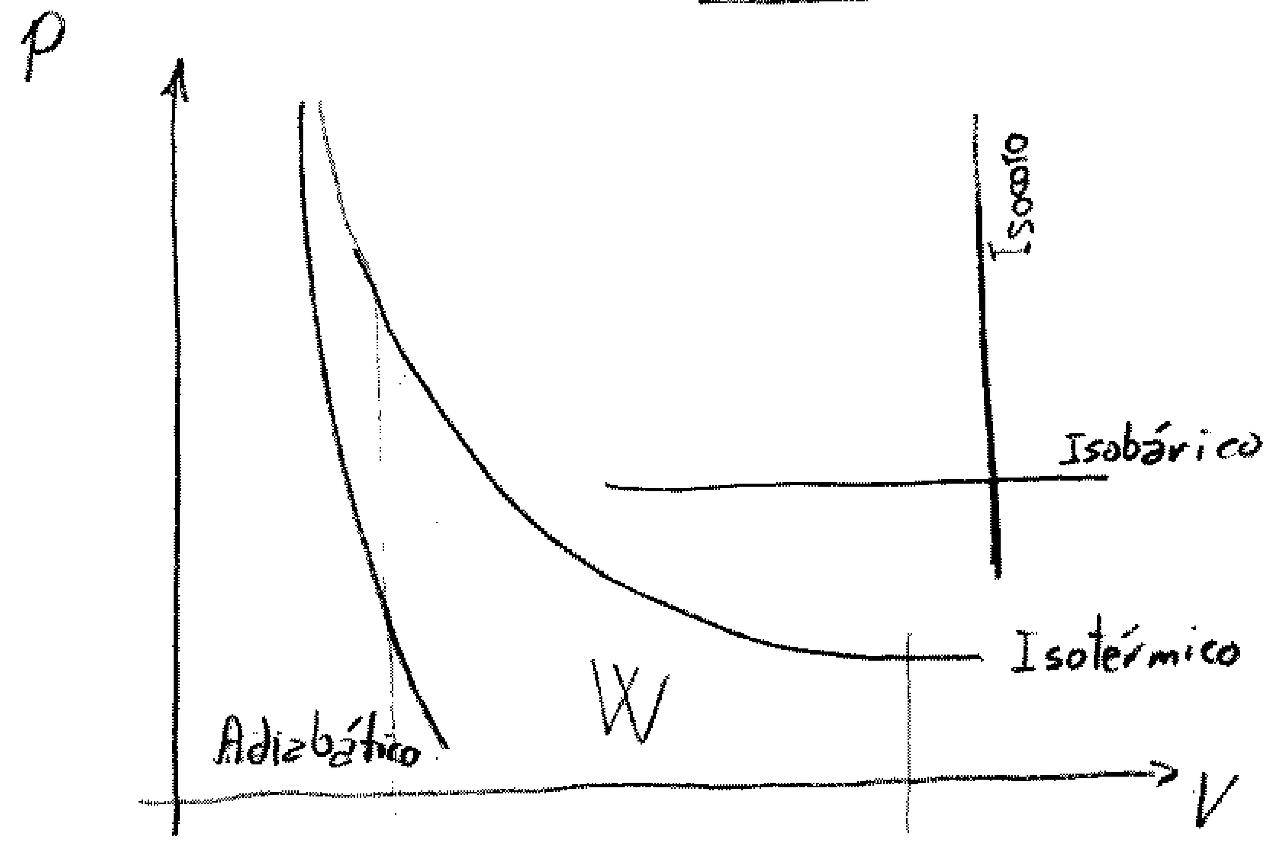
Autoenergía: $\boxed{U = \vartheta N \frac{1}{2} KT = \frac{\vartheta nRT}{2}}$

$\vartheta \rightarrow$ gr. libertad accesibles

- Gases monoatómicos $\rightarrow 3$
  - Bajas Temp $\rightarrow 3$
  - Temp. amb $\rightarrow 5$
  - Altas Temp. $\rightarrow 7$

$\boxed{Q = \Delta U + W}$ $\boxed{W_{ab} = \int_a^b P\,dV = nRT_a \ln\left(\frac{V_b}{V_a}\right)}$ (para T=cte)

$1Pa = 1 N/m^2$
$1atm = 1'013 \cdot 10^5\ Pa$
$1bar = 10^5\ Pa$



Isotérmico $\rightarrow T = cte\ (U = cte) \rightarrow Q = W$

Isocoro $\rightarrow V = cte\ (W = 0)$

Isobárico $\rightarrow P = cte$ — $W = P\Delta V$

Adiabático $\rightarrow Q = \cancel{cte}\ 0$ $\boxed{PV^\gamma = cte}$ $\boxed{\gamma = \frac{\vartheta + 2}{\vartheta}} = \frac{C_P}{C_V}$ — Calor a P cte/V cte

$\boxed{C_V = \left.\frac{Q}{\Delta T}\right|_{V=cte} = \frac{\Delta U}{\Delta T} = \frac{\vartheta R}{2}}$

$\boxed{C_P = \left.\frac{Q}{\Delta T}\right|_{P=cte} = \frac{\Delta U + W}{\Delta T} = C_V + R} = \frac{\vartheta R}{2} + R = R\left(1 + \frac{\vartheta}{2}\right) = R\left(\frac{2+\vartheta}{2}\right)$

Ondas

$\boxed{y(x,t) = A \sin(\omega t + Kx)}$

$\omega = 2\pi f$; $K = \frac{2\pi}{\lambda}$; $v = \lambda f$

- Ondas transversales en la cuerda / Acústicas

$\boxed{V_p = \sqrt{\frac{T}{\mu}}} = \frac{K}{\omega^2}$; $\mu = \frac{\text{masa}}{\text{longitud}} \rightarrow \rho$ (sobrepresión acústica) ; $\boxed{V_p = \omega \cdot y_0}$; $\boxed{Z_0 = \sqrt{T\mu}} \rightarrow \sqrt{B\rho}$

Acústica: E/unid V

$\langle \eta \rangle = \boxed{K\frac{\Delta E_x}{\Delta x}} = \langle \frac{\Delta U}{\Delta x} \rangle = \frac{1}{4}\mu(\omega y_0)^2 = \frac{1}{4} F (K y_0)^2 = \boxed{\frac{1}{2}\mu V_p^2} \rightarrow \frac{1}{2}\rho V_p^2$

$\boxed{\langle P \rangle = \frac{1}{2}\sqrt{T\mu}\, V_0^2 = \frac{1}{2} Z_0 V_p^2}$

sobrepresión $\rightarrow \boxed{\Delta p = -\rho \frac{\Delta V}{V} = B\frac{\Delta V}{V}}$ $\boxed{B = \gamma P}$ $\boxed{V_p = \sqrt{\frac{\gamma RT}{M_{mol}}}}$

Analogías Cuerda/Acústica
$\mu = \rho$
$T = B$

$\boxed{E = \frac{1}{2} KA^2}$ $\boxed{P = \frac{E}{\Delta t}}$ $\boxed{I = \frac{P}{S}}$ $\boxed{I = I_0 e^{-\beta R}}$

$\boxed{y_r = \underbrace{2A\cos\left(K\frac{r'-r}{2}\right)}_{cte} \sin\left(\omega t - K\frac{r'+r}{2}\right)} = A_r \cos\left(\omega t - K\frac{r'+r}{2}\right)$

Interf: Constructiva $\Leftrightarrow r' - r = n\lambda$; Destructiva $\Leftrightarrow r' - r = (2n+1)\frac{\lambda}{2}$

Oscilaciones

$$\overbrace{m\frac{d^2x}{dt^2}}^{F_{masa}} + \overbrace{b\frac{dx}{dt}}^{F_{rozamiento}} + \overbrace{kx}^{F_{recuperadora}} = F_{ext} \Rightarrow \frac{d^2x}{dt} + \frac{1}{\tau}\frac{dx}{dt} + \omega_0^2 x = \frac{F_{ext}}{m}$$

- MAS → $b=0$; $\tau=\infty$
- Osc. Lib. Amor → $F_{ext}=0$
- Osc. Forzado

- Analogías Mecánico - Eléctrico

$$\left[\begin{array}{ll} x \to q & m \to L \\ v \to i\,(A) & b \to R \\ F \to V\,(V) & k \to 1/C \end{array}\right]$$

$$\boxed{\tau = \frac{m}{b} = \frac{L}{R}} \qquad \boxed{\omega_0^2 = \frac{k}{m} = \frac{1}{LC}}$$

- (MAS)

$$\boxed{x(t) = A_0 \sin(\omega_0 t + \varphi_0) = A e^{i(\omega_0 t + \varphi_0)}}$$

$$\boxed{E = U + E_K = \frac{1}{2}\overbrace{m\omega_0^2}^{K} A^2} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

- Oscilador Libre Amortiguado

$$\boxed{x(t) = Ae^{-t/2\tau}\sin(\omega_d t + \varphi(0))} \qquad \boxed{\omega_d = \omega_0\sqrt{1-\frac{1}{4Q^2}}} \qquad \boxed{Q = \omega_0\tau} \approx \omega_d\tau$$

$$\boxed{E(t) \approx \frac{1}{2}kA^2 e^{-t/\tau}} = E(0)e^{-t/\tau}$$

- Oscilador Forzado (Régimen permanente sinusoidal)

$$\boxed{\begin{array}{l} x_L(t) = Ae^{-\frac{t}{2\tau}}\sin(\omega_d t + \varphi(0)) \\ x_p(t) = \frac{V_0}{\omega}\sin(\omega t + \varphi(v)) \end{array}}$$

$x(t) = x_p(t) + x_L(t)$

CI → $x(0) = x_p(0) + x_L(0)$

$\dot{x}(0) = v_p(0) + v_L(0)$

$$\boxed{Z(\omega) = b + j\left(\omega m - \frac{k}{\omega}\right)}$$

Resonancia ⇒ $\boxed{\omega_0 = \sqrt{\frac{k}{m}}}$

característica

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}} = \sqrt{Km}$$

$$F(t) = F_0 e^{i\omega t} = m\frac{dv}{dt} + bv + k\int v\,dt \Rightarrow \boxed{F_0 = mv_1(j\omega) + bv_1 + \frac{kv_1}{j\omega} = v_1 Z(\omega)}$$

$v(t) = v_0 e^{i(\omega t + \varphi_v)} = v_1 e^{i\omega t}$

$$\boxed{v_0 = |v_1| = \frac{F_0}{|Z(\omega)|}} \ ; \ \varphi = -\arg[Z(\omega)] \qquad \boxed{x_0 = \frac{V_0}{\omega}}$$

Ancho de Banda ⇒ $\boxed{\Delta\omega = \frac{1}{\tau}}$

E_{BOBINA} $\boxed{\langle E_K(\omega)\rangle = \frac{1}{4}mv_0^2(\omega)}$

$E_{CONDENS}$ $\boxed{\langle U(\omega)\rangle = \frac{1}{4}Kx_0^2(\omega)}$

$$\boxed{\frac{\langle E_K(\omega)\rangle}{\langle U(\omega)\rangle} = \left(\frac{\omega}{\omega_0}\right)^2} \qquad \boxed{\langle P\rangle = \frac{1}{2}bv_0^2} = \frac{1}{2}Ri^2$$

- Otras fórmulas útiles

$$Z(\omega) = Z(\omega_0) + jZ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$