

ANTENAS

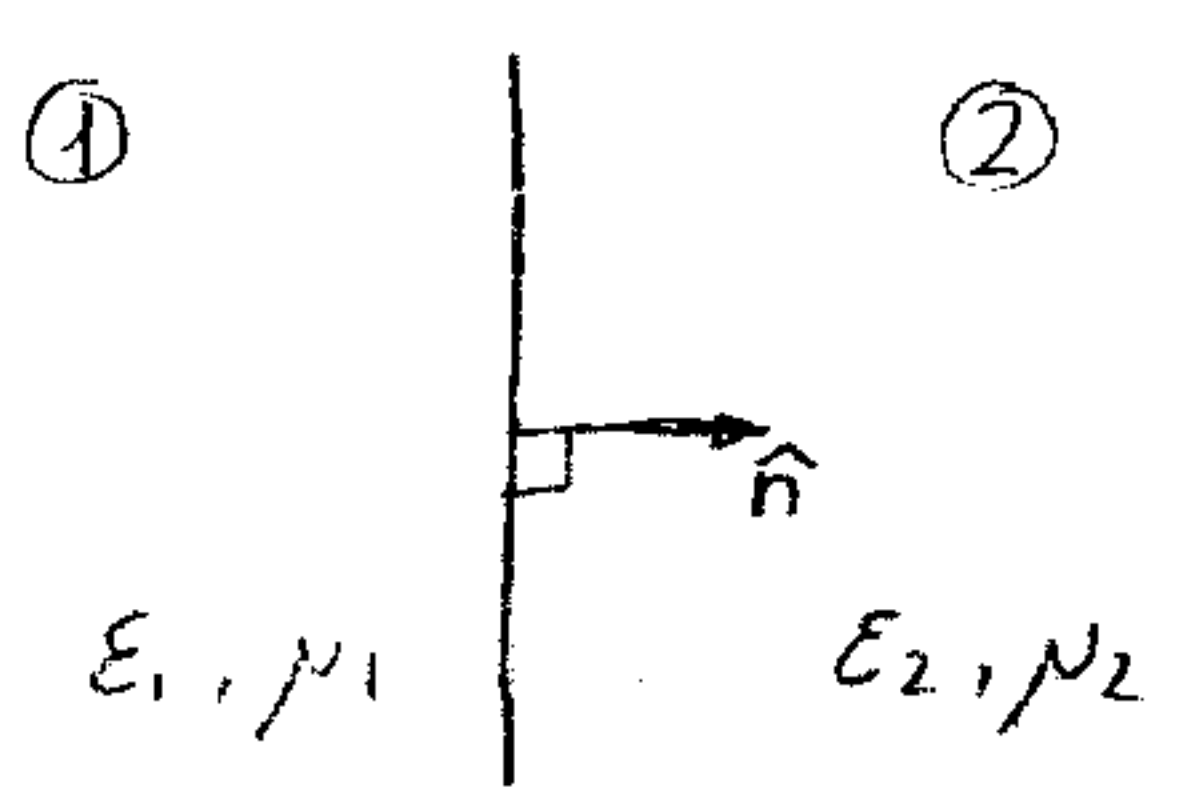
1-1: FUNDAMENTOS DE RADIACIÓN

• Ecs. Maxwell

| FORMA INTEGRAL | DIFERENCIAL | FASORIAL |
|---|--|--|
| $\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{S}$ | $\nabla \times \vec{E} = -\frac{d}{dt} \vec{B}$ | $\nabla \times \vec{E} = -j\omega \mu \vec{H}$ |
| $\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S} + \frac{d}{dt} \int_S \vec{D} \cdot d\vec{S}$ | $\nabla \times \vec{H} = \vec{J} + \frac{d}{dt} \vec{D}$ | $\nabla \times \vec{H} = \vec{J} + j\omega \epsilon \vec{E}$ |
| $\int_S \vec{D} \cdot d\vec{S} = \int_V \rho \, dv$ | $\nabla \cdot \vec{D} = \rho$ | $\nabla \cdot \vec{E} = \rho / \epsilon$ |
| $\oint_S \vec{B} \cdot d\vec{S} = 0$ | $\nabla \cdot \vec{B} = 0$ | $\nabla \cdot \vec{H} = 0$ |
| $\oint_S \vec{J} \cdot d\vec{S} = -\frac{d}{dt} \int_V \rho \, dv$ | $\nabla \cdot \vec{J} = -\frac{d}{dt} \rho$ | $\nabla \cdot \vec{J} = -j\omega \rho$ |

• Notación fasorial $\Rightarrow \vec{J}(\vec{r}, t) = \text{Re}\{\vec{J}(\vec{r})e^{-j\omega t}\}$

• Condiciones de contorno



$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = -\vec{M}_s$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}$$

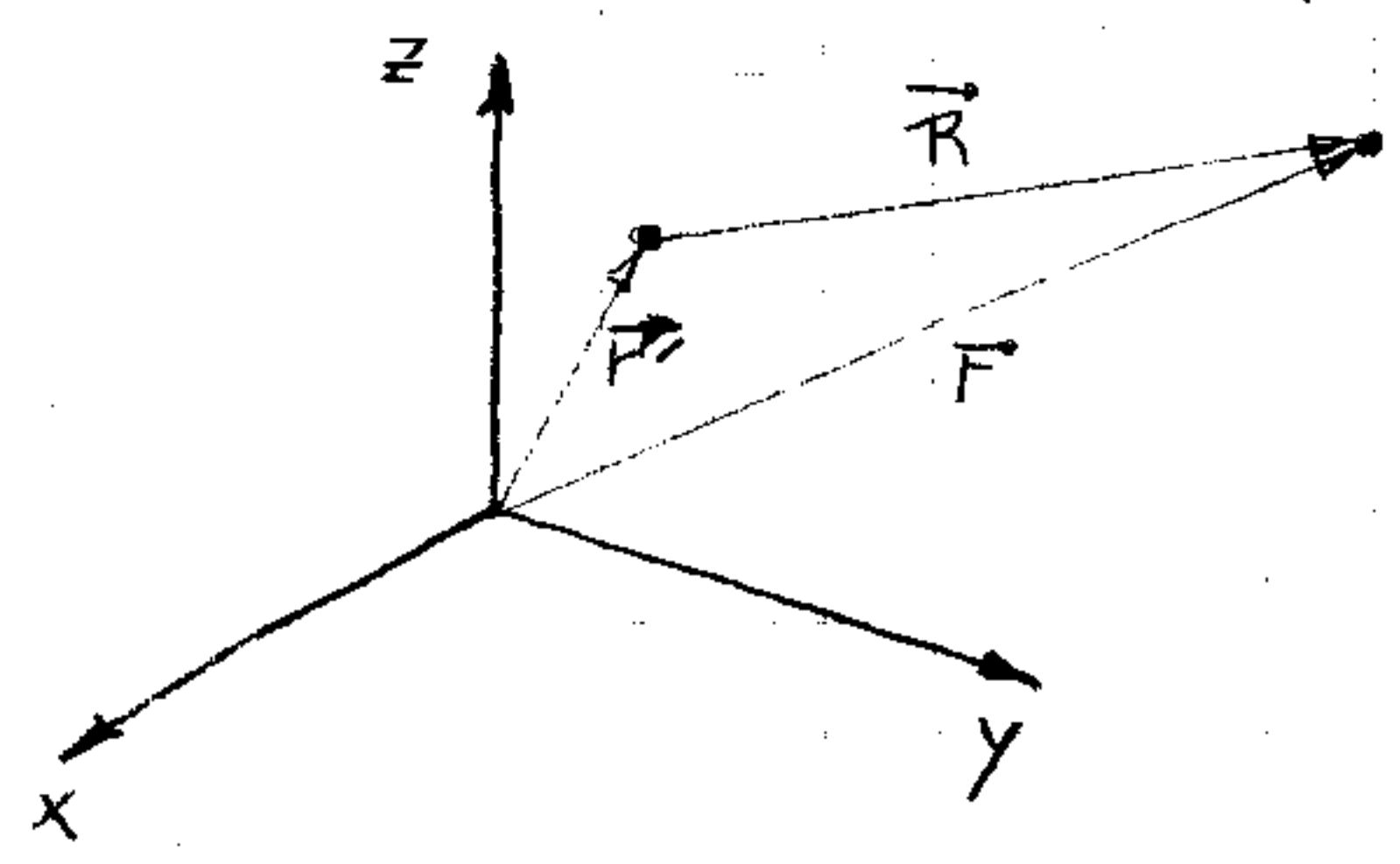
Dens. corriente magnet

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \sigma_s$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = \tau_s$$

Dens. vol. carga magnet

• Expresiones de los campos



$$\varphi(\vec{r}) = \frac{1}{4\pi\epsilon} \int_{V'} \rho(\vec{r}') \frac{e^{-jkR}}{R} \, dv'$$

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_{V'} \vec{J}(\vec{r}') \frac{e^{-jkR}}{R} \, dv'$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla \cdot \varphi - j\omega \vec{A}$$

\vec{r} : punto campo
 \vec{r}' : punto fuente

$$\vec{R} = \vec{r} - \vec{r}'$$

◦ Aprox. a grandes distancias para campos radiados

$$\vec{N} = \int_{V'} \vec{J}(\vec{r}') e^{jk\hat{r}\vec{r}'} dV'$$

↳ Vector de Radiación

$$\vec{A} = \frac{\mu}{4\pi} \cdot \frac{e^{-jk\hat{r}r}}{r} \vec{N}$$

$$\vec{E} = j\omega \hat{r} \times (\hat{r} \times \vec{A}) \quad \vec{H} = -\frac{j\omega}{\eta} \hat{r} \times \vec{A}$$

$$\vec{E} = \eta (\vec{H} \times \hat{r}) \quad \vec{H} = 1/\eta (\hat{r} \times \vec{E})$$

$$\vec{P} = \text{Re} \{ \vec{E} \times \vec{H}^* \} = \eta |\vec{H}|^2 \hat{r} = 1/\eta |\vec{E}|^2 \hat{r} \quad \text{Vector de Poynting}$$

$$P_r = \int_S \vec{P} \cdot d\vec{S} = \int_S \vec{P} \cdot \hat{r} r^2 \sin\theta d\theta d\phi \quad \text{Pot. radiada}$$

$$t(\theta, \phi) = \frac{P(\theta, \phi)}{P_{\max}} = \frac{|\vec{E}_\theta|^2 + |\vec{E}_\phi|^2}{[|\vec{E}_\theta|^2 + |\vec{E}_\phi|^2]_{\max}} = \frac{|\vec{H}_\theta|^2 + |\vec{H}_\phi|^2}{[|\vec{H}_\theta|^2 + |\vec{H}_\phi|^2]_{\max}} \quad \text{Diagrama de Radiación}$$

$$D(\theta, \phi) = \frac{P(\theta, \phi)}{P_r / 4\pi r^2} = D \cdot t(\theta, \phi) \quad D = \frac{4\pi}{\lambda^2} A_{\text{ef}} \approx \frac{4\pi}{\Delta\theta \Delta\phi}$$

| | | | |
|-------------------|--------------------------|-------------------------------------|-------------------------|
| $R_r = P_r / I^2$ | Resistencia de Radiación | $\eta = \frac{R_r}{R_r + R_\Omega}$ | Eficiencia de Radiación |
|-------------------|--------------------------|-------------------------------------|-------------------------|

$$l_{\text{ef}} = \frac{V_{\text{ca}}}{|\vec{E}|}$$

$$A_{\text{ef}} = \frac{P_{\text{rad}}}{P} = \frac{V_{\text{ca}}^2 \eta}{4R_r |\vec{E}|^2} = \frac{l_{\text{ef}}^2 \eta}{4R_r}$$

◦ Zonas de Fresnel y Fraunhofer

* Z. Fraunhofer $\Rightarrow \frac{2D^2}{\lambda} \leq r \leq \infty$ (campo lejano)

* Z. Fresnel $\Rightarrow 0.6 \sqrt{\frac{D^3}{\lambda}} \leq r \leq \frac{2D^2}{\lambda}$

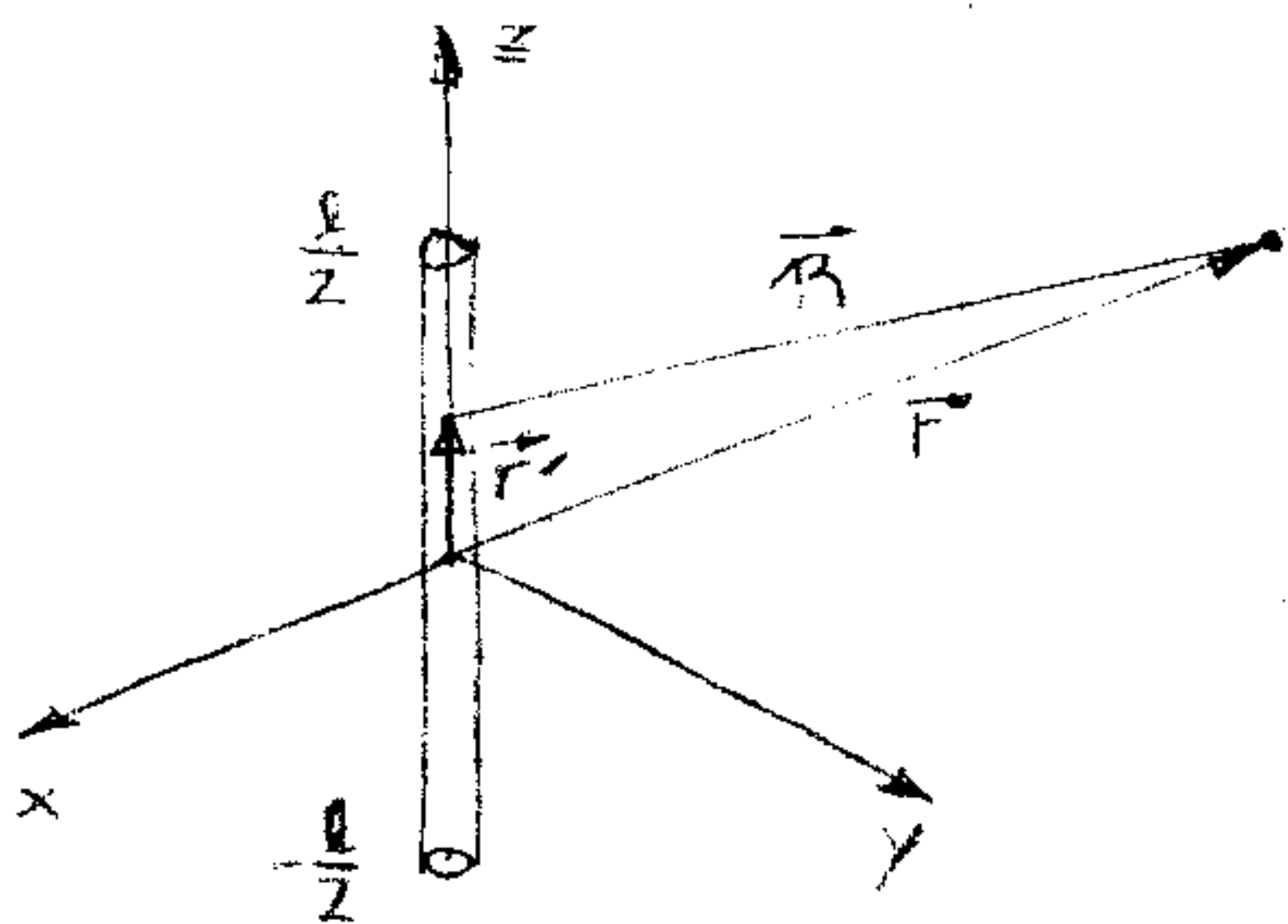
Dependiendo de la zona tendremos que coger más términos de la aprox. de R

$$R = |\vec{r} - \vec{r}'| = \left\{ \text{Si } \vec{r}' = z' \hat{z} \right\} = r - z' \cos\theta + \frac{z'^2}{2r} \sin^2\theta + \dots$$

1-2 : ANÁLISIS DE ANTENAS BÁSICAS

◦ DIPOLO ELEMENTAL ELECTRICO

$$l \ll \lambda$$



$$\vec{J}(\vec{r}') = I \delta(x') \delta(y') \quad |z'| \leq l/2$$

$$\vec{N} = I l \hat{z}$$

Aprox. para dip. eléctricos
(mirar más abajo)

$$H_r = H_\theta = 0$$

$$E_r = E_\phi = 0$$

$$H_\phi = j \frac{k I l}{4\pi r} e^{-jkr} \sin\theta$$

$$E_\theta = j \frac{k \eta I l}{4\pi r} e^{-jkr} \sin\theta$$

$$P_r = \frac{2}{3} \pi \eta I^2 \left(\frac{l}{\lambda}\right)^2$$

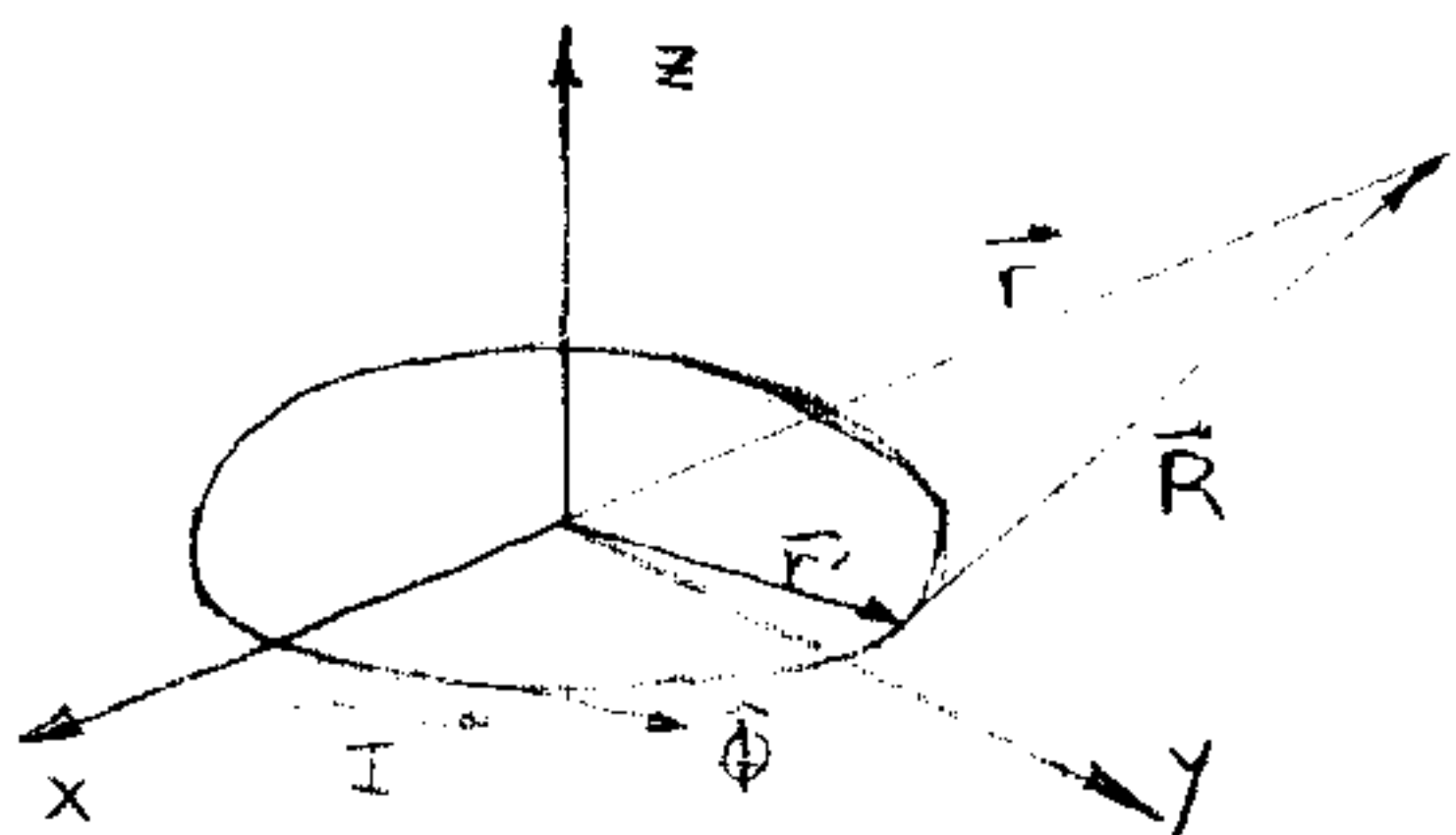
$$R_r = 80 \pi^2 \left(\frac{l}{\lambda}\right)^2$$

$$D(\theta, \phi) = 1.5 \sin^2 \theta$$

$$l_g = l$$

◦ ESPIRA CIRCULAR ELEMENTAL

$$a \ll \lambda$$



$$\vec{N} = j \pi k a^2 I \sin\theta \hat{\phi}$$

$$E_r = E_\theta = 0$$

$$E_\phi = k^2 \eta \pi a^2 I \frac{e^{-jkr}}{4\pi r} \sin\theta$$

$$P_r = \frac{\pi \eta}{6} (ka)^2 I^2$$

$$R_r = 20 \pi^2 (ka)^4$$

$$D(\theta, \phi) = 1.5 \sin^2 \theta$$

- Expresiones generales del vector radiación (antenas elementales)

◦ Dipolos eléctricos

$$l \ll \lambda$$

(Para dipolos)



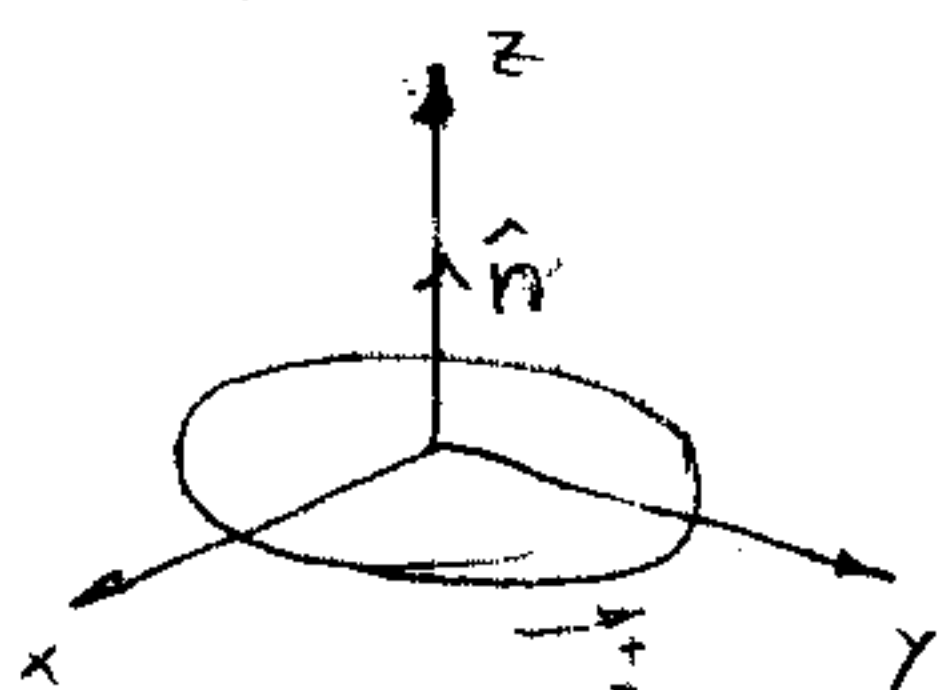
$$\vec{N} = \int_V \vec{J}(\vec{r}') dV' = j \omega \vec{p}$$

$$\vec{p} = \int_V \rho(\vec{r}') \vec{r}' dV'$$

◦ Dipolos magnéticos

$$a \ll \lambda$$

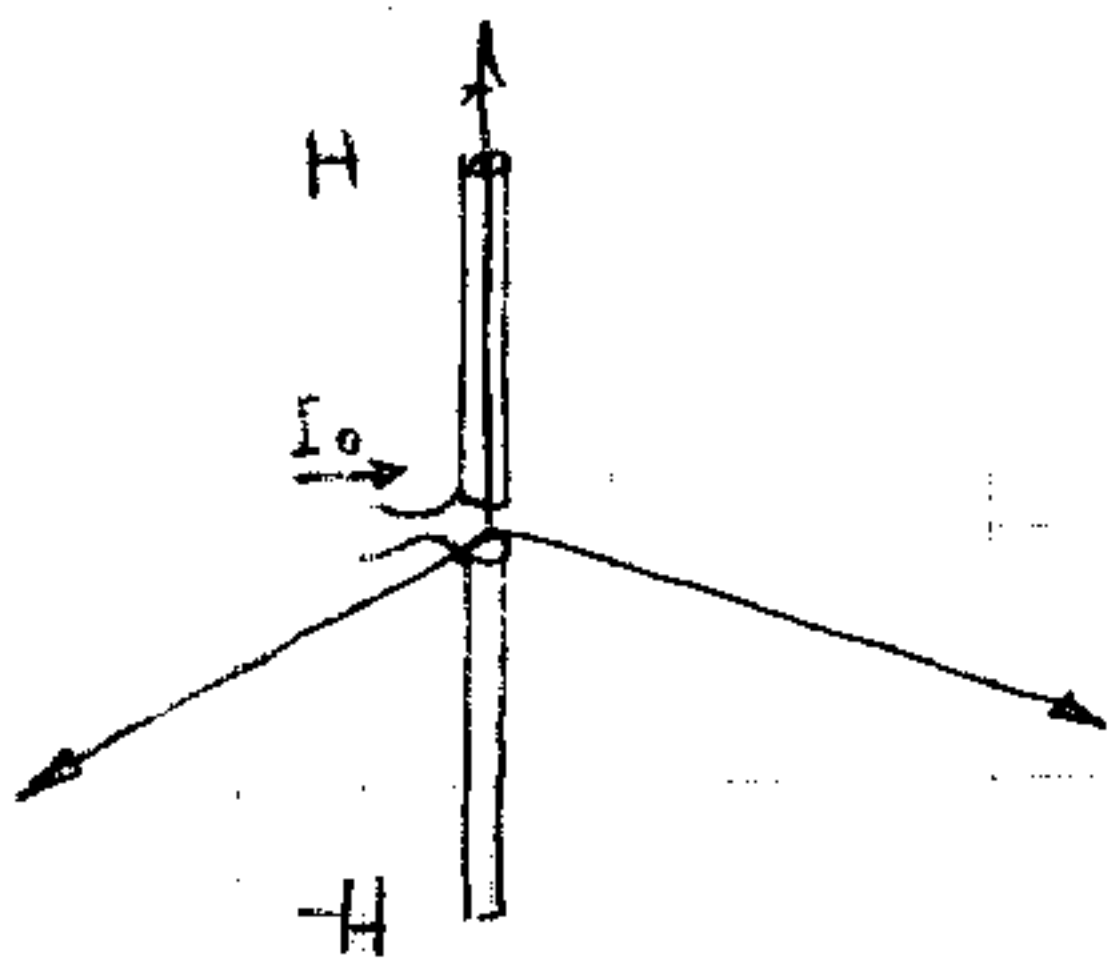
(Para espiras, de cualquier geomet)



$$\vec{N} = j k \vec{m} \times \vec{r}$$

$$\vec{m} = I \cdot \hat{n} \cdot \text{Area}$$

• ANTENAS CILÍNDRICAS



$$I(z') = \begin{cases} I_m \sin k(H - |z'|) & |z'| \leq H \\ 0 & |z'| \geq H \end{cases}$$

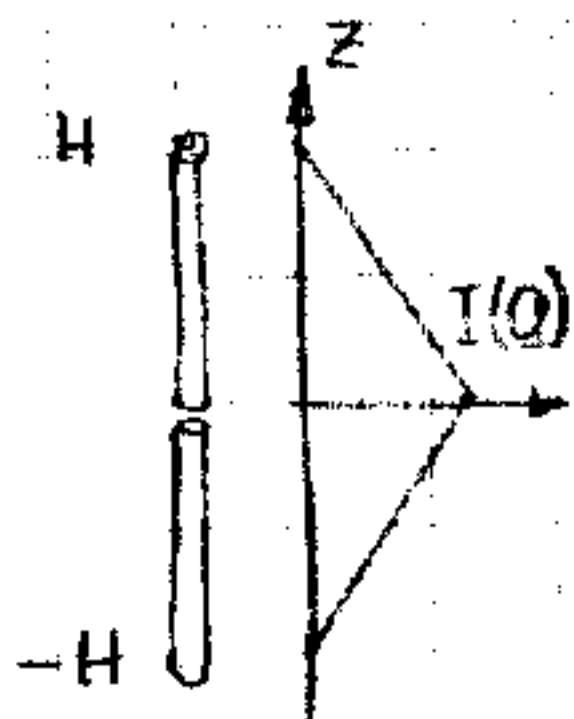
Resolución numérica \Rightarrow Gráfica R_r, D

El resto de parámetros se obtienen a partir de estos

- Dipolo corto

$$H < 0.1 \lambda$$

$$l_{eq} = l/2$$



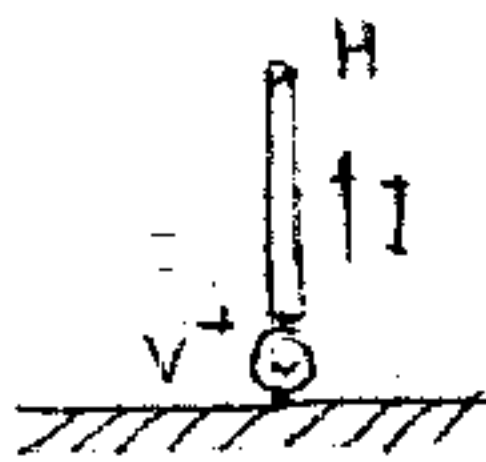
$$\vec{N} = H \cdot I(0) \hat{z} = \frac{I(0) \cdot l}{2} \hat{z} = \frac{\vec{N}_{elemental}}{2}$$

$$|\vec{E}| = \frac{1}{2} |\vec{E}|_{elem}$$

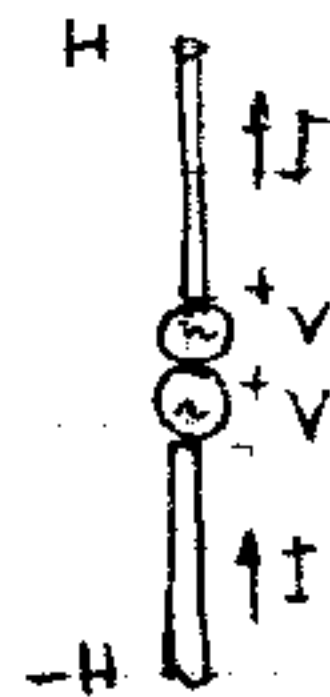
$$P = \frac{P_{elem}}{4}$$

$$R_r = \frac{1}{4} R_{r,elem}$$

- Monopolo con plano de masa



Teoría de Imágenes



$$P_{mono} = \begin{cases} P_{dipolo} & z > 0 \\ 0 & z < 0 \end{cases}$$

$$P_{r,mono} = P_{r,dipolo}$$

$$D_{mono} = 2 D_{dipolo}$$

$$A_{ef,mono} = \frac{1}{2} A_{ef,dipolo}$$

- Resistencia de entrada de antenas cilíndricas

$$Z_i = -j 120 \left[\ln\left(\frac{2H}{a}\right) - 1 \right] \cotg kH$$

Es capacitiva \Rightarrow No resonante ($X_i \neq 0$)



- long. Equivalente

$$l_{eq} = \frac{1}{I(0)} \int_{-H}^H I(z) dz$$

◦ Dip. Elemental

$$l_{eq} = l$$

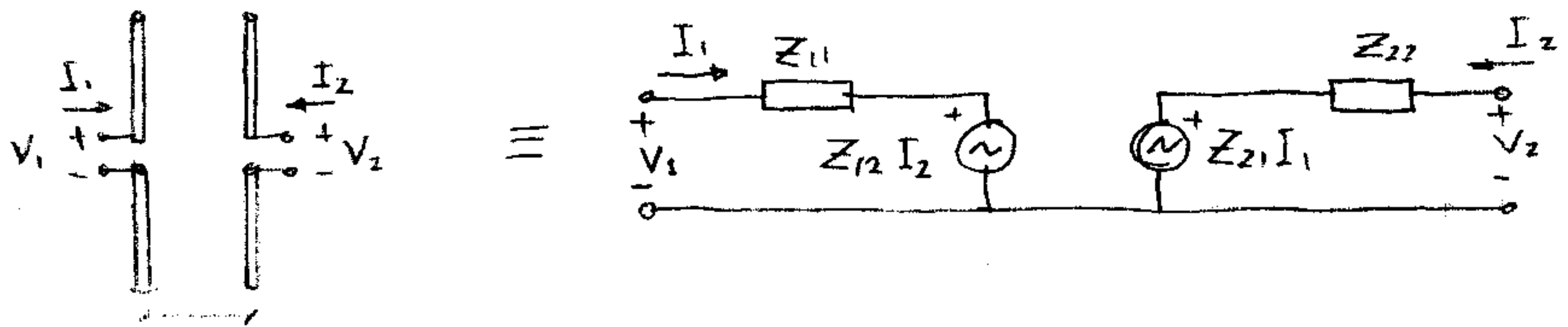
◦ Dip. Corto

$$l_{eq} = l/2 = H$$

◦ Dip. en $\lambda/2$

$$l_{eq} = l/\pi$$

TEOREMA DE RECIPROCIDAD



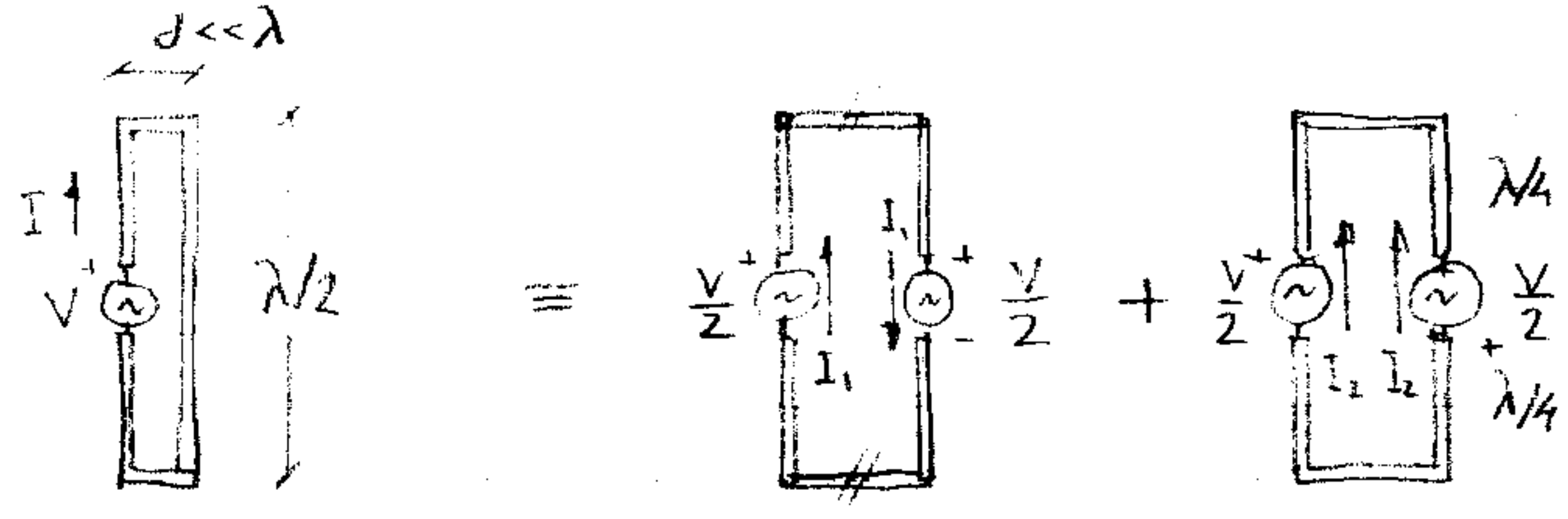
Reciprocidad \Rightarrow $Z_{12} = Z_{21}$ $\frac{P_{R_{x1}}}{P_{T_{x2}}} = \frac{P_{R_{x2}}}{P_{T_{x1}}}$

- Antenas idénticas $\Rightarrow Z_{11} = Z_{22}$
- la impedancia de una antena en recepción y en emisión es la misma.

$$\begin{aligned} V_1 &= I_1 Z_{11} + I_2 Z_{12} \\ V_2 &= I_1 Z_{21} + I_2 Z_{22} \end{aligned}$$

la impedancia mutua en función de la distancia se obtiene a partir de gráficas.

Dipolo doblado



$$Z_{in} = 4 Z_{11}$$

BW_{dip.dobl} >> BW_{dip}

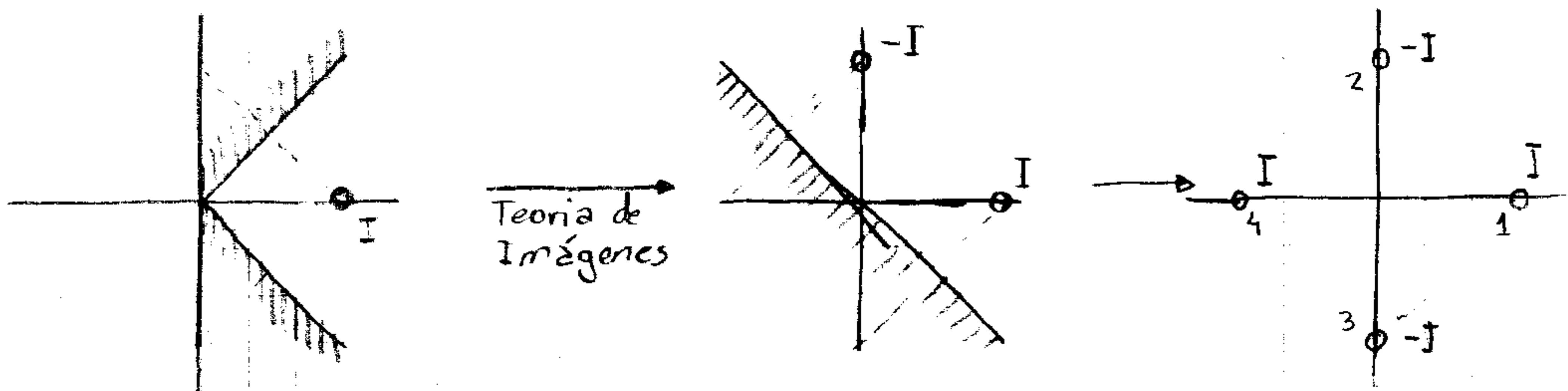
L.T. $\lambda/4 \Rightarrow c.o$
 $I_2 = 0$

Coefficiente de desacoplo por polarización

$$C_p = |\hat{e}_r \cdot \hat{e}_t|^2$$

- $\vec{E}(\vec{r}) = C (\hat{x} \pm j\hat{y}) e^{-jkz}$ Pol. circular
pol. izq (up arrow), pol. dch (down arrow)
- $\vec{E}(\vec{r}) = C (\hat{x} \pm b\hat{y}) e^{-jkz}$ Pol. lineal
- $\vec{E}(\vec{r}) = C (\hat{x} \pm (1+jb)\hat{y}) e^{-jkz}$ Pol. elíptica

- Receptor diédrico



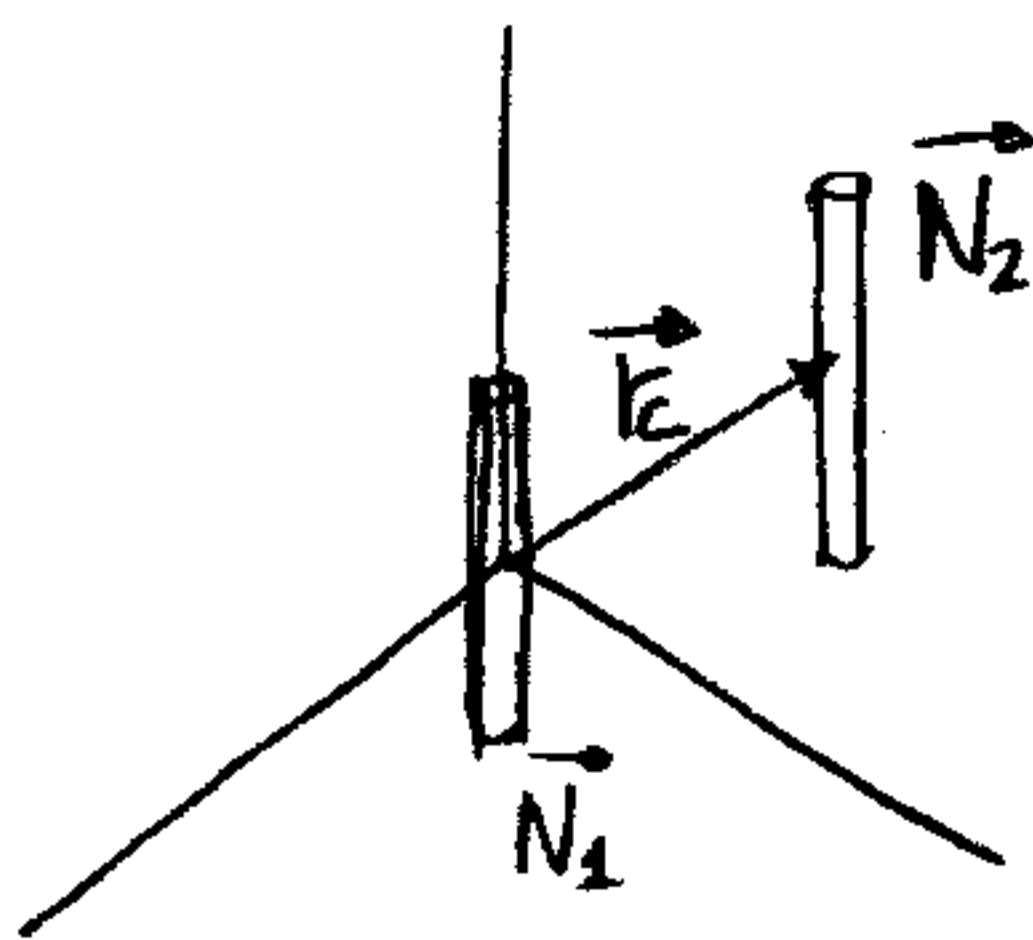
$$V_1 = I_1 Z_{11} + I_2 Z_{12} + I_3 Z_{13} + I_4 Z_{14}$$

$$I_1 = -I_2 = -I_3 = I_4$$

$$Z_{in} = \frac{V_1}{I_1} = Z_{11} - 2Z_{12} + Z_{14}$$

↑
 $Z_{12} = Z_{13}$

- Antenas no dispuestas en el origen de coordenadas



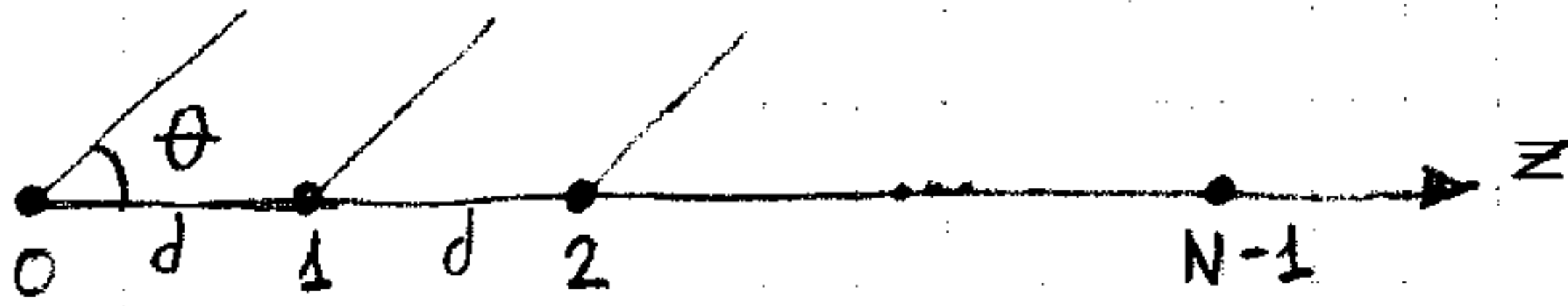
$$\vec{N}_2 = \vec{N}_1 e^{j\vec{k} \cdot \vec{r}_c}$$

- Directores y reflectores

Se puede analizar por el teorema de reciprocidad tomando la tensión de los elementos no alimentados nula para obtener la intensidad que circula por ellos y a continuación superponer los campos producidos por cada uno (Aunque el tamaño de los elementos sea distinto se supone igual.)

T-3 AGRUPACIONES DE ANTENAS

- Factor de agrupación (Antenas iguales equiespaciadas)



$$\vec{E} = \vec{E}_0 (I_0 + I_1 e^{jkdcos\theta} + \dots + I_{N-1} e^{jK(N-1)d cos\theta})$$

$$I_n = a_n e^{jn\alpha} \quad (\text{fase progresiva})$$

$$\vec{E} = \vec{E}_0 \cdot AF(\psi)$$

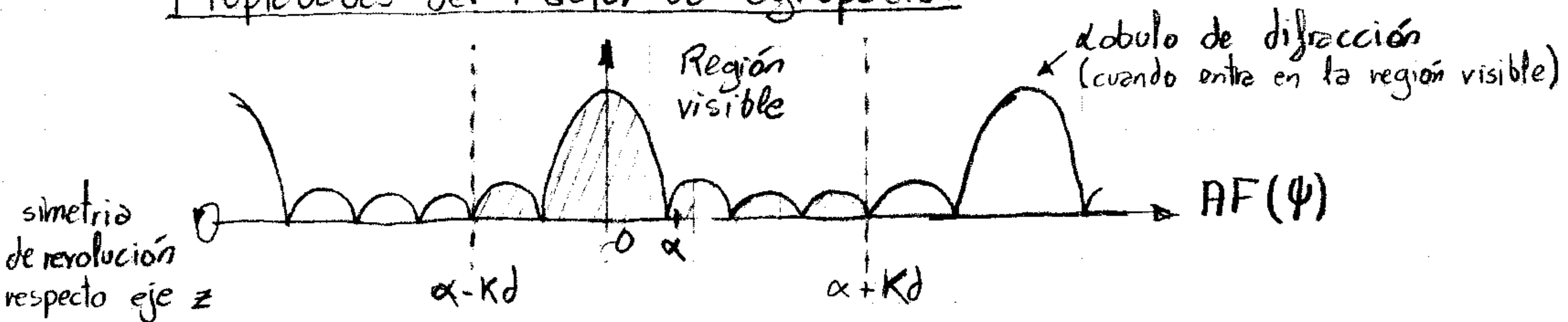
$$AF(\psi) = \sum_{n=0}^{N-1} a_n e^{jn\psi}$$

Factor de agrupación

Ángulo eléctrico

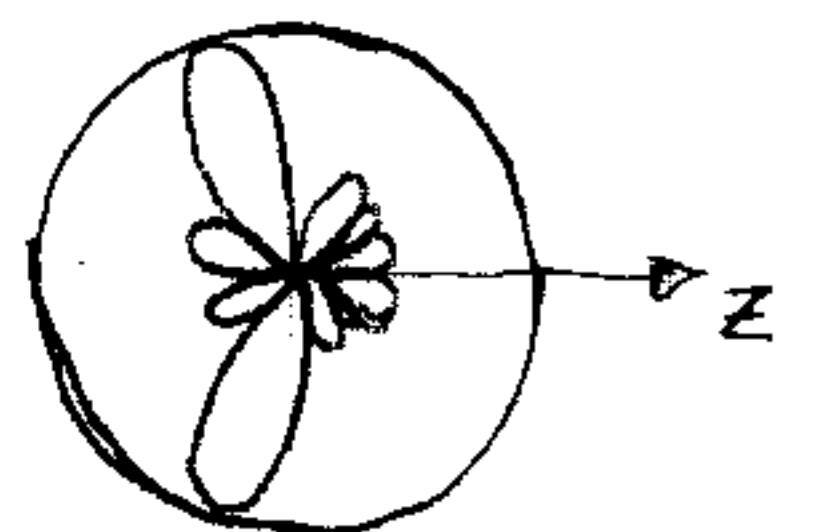
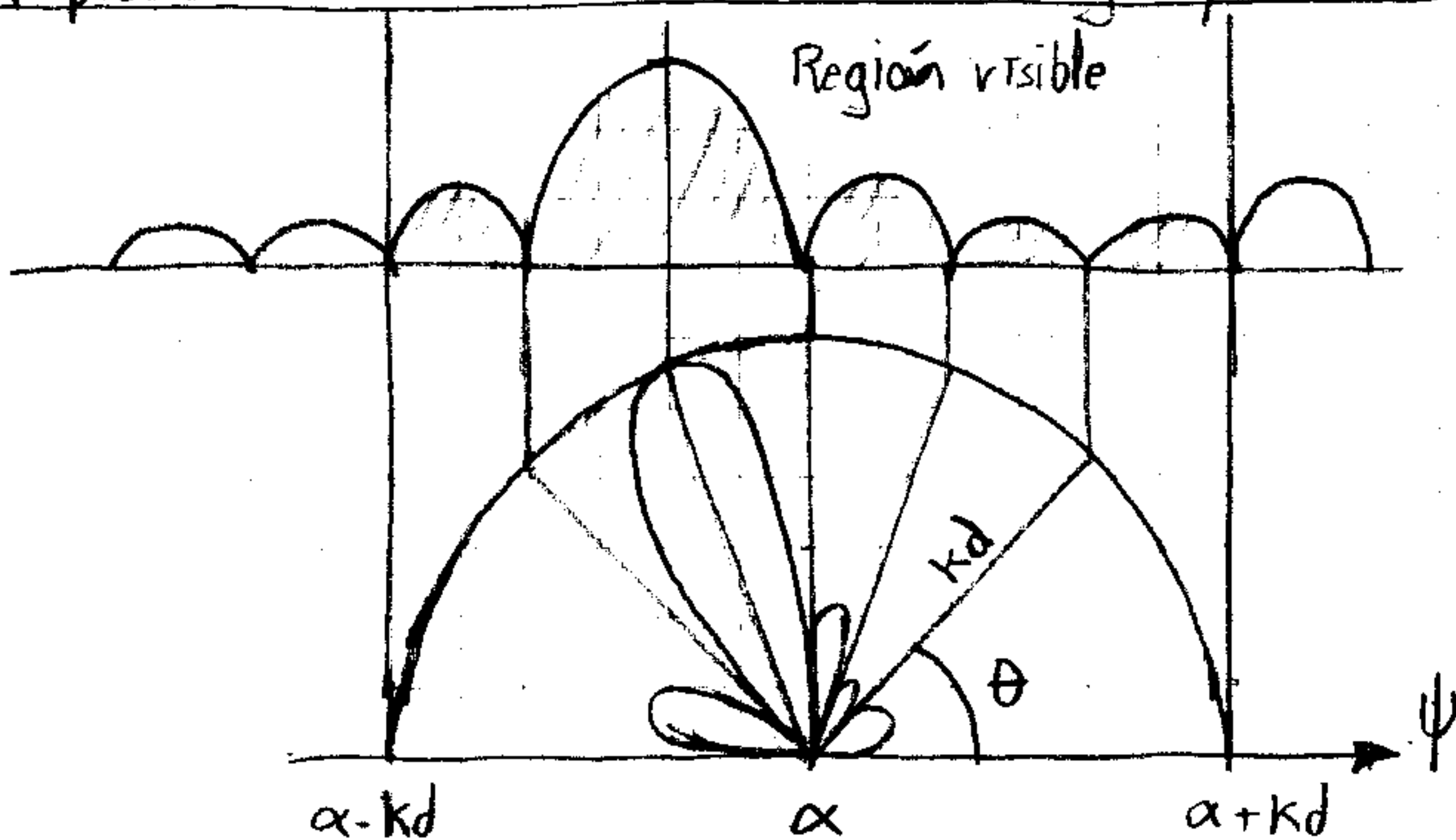
$$\psi = Kd \cos \theta + \alpha$$

- Propiedades del Factor de agrupación



- Tiene periodo 2π
- Se corresponde con la DFT de los coef. a_n
- Si $a_n > 0$, el máximo se encuentra en $\psi = 0$,

- Representación del Factor de agrupación



- Polinomio de agrupación

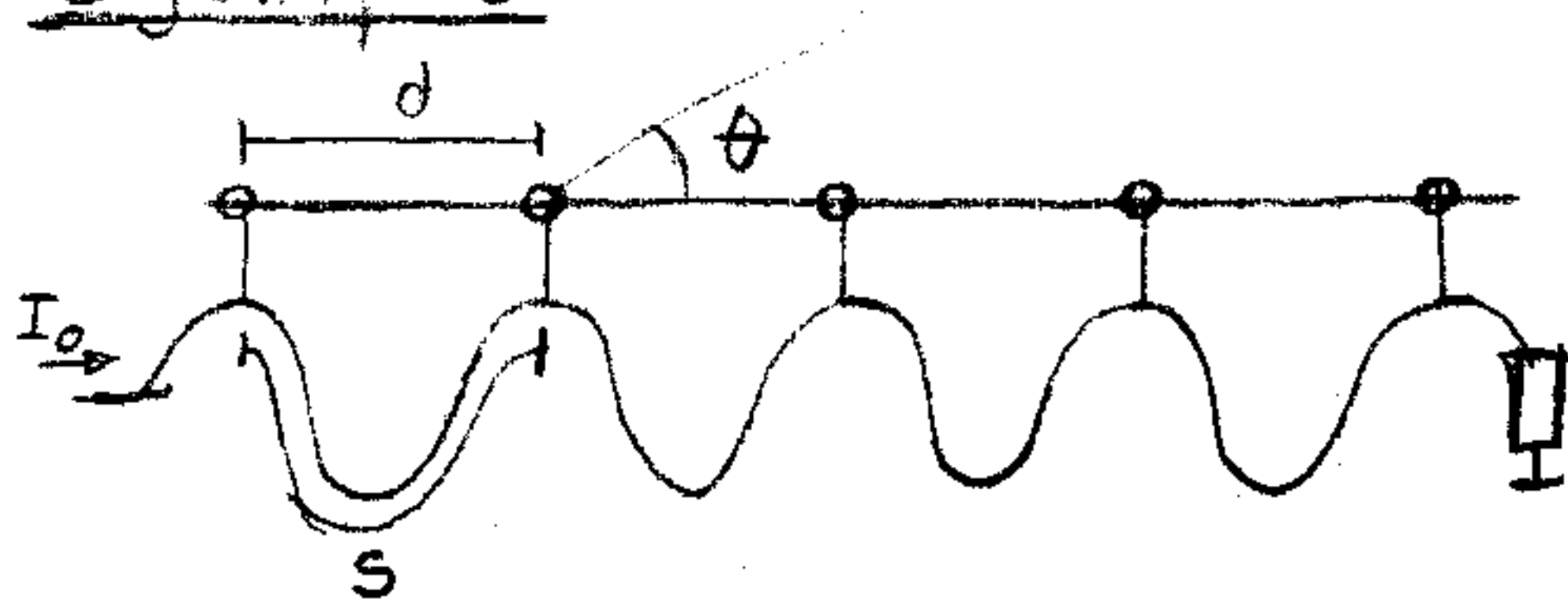
• Transformada z de los coef. de la agrupación

$$P(z) = \sum_{n=0}^{N-1} a_n z^{-n}$$

$$AF(\psi) = P(z) \Big|_{z=e^{j\psi}}$$

$$LSR = 20 \log \frac{|AF(\psi_{MAX})|}{|AF(\psi_{MAX2})|}$$

- Ejemplo:



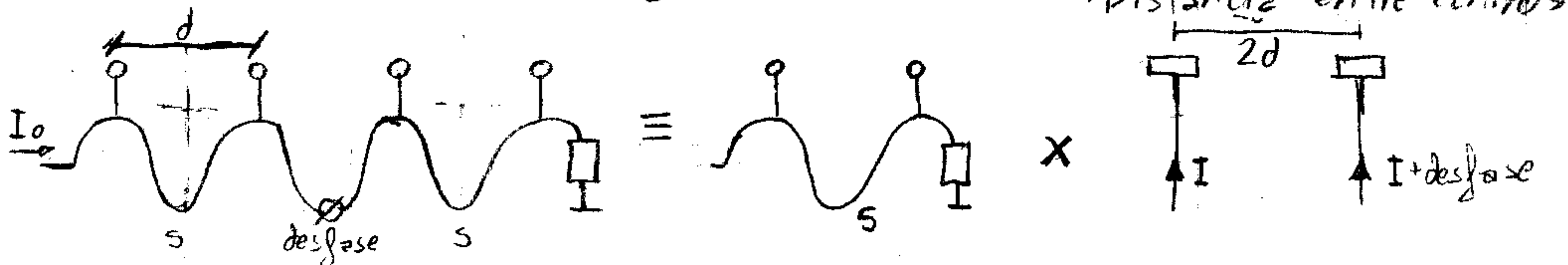
$$kd = \frac{2\pi}{\lambda} d$$

$$\alpha = -ks$$

- Dirección del máximo

$$kd \cos \theta_{max} + \alpha = \psi \Big|_{AF_{max}}$$

- Analisis como varias agrupaciones



El AF total es la multiplicación de ambos AF (en forma de diagrama de radiación)

- Antenas con distribución uniforme

$$P(z) = 1 + z + z^2 + \dots + z^{N-1} = \frac{z^N - 1}{z - 1}$$

$$|AF(\psi)| = \left| \frac{\sin\left(N \frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \right|$$

$$\psi_c = n \frac{2\pi}{N} \quad (\text{ceros})$$

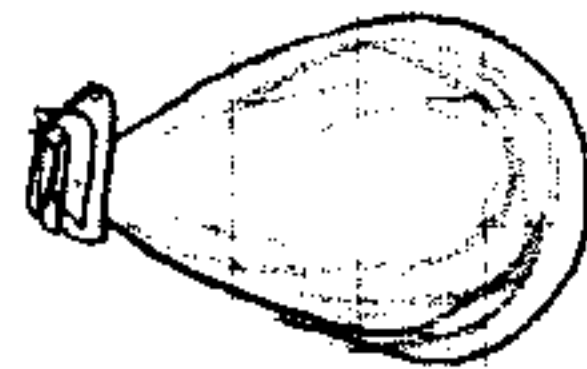
$$SLR = N \left| \sin\left(\frac{3\pi}{2N}\right) \right| \Rightarrow \lim_{N \rightarrow \infty} SLR = 13'2 \text{ dB}$$

o Broadside ($\alpha = 0^\circ$, $\theta_{max} = 90^\circ$)

$$\Delta_{-3dB} \approx 0'88 \frac{\lambda}{Nd}$$

o Endfire ($\alpha = \pm \pi d$)

$$\Delta_{-3dB} \approx \sqrt{3'5} \frac{\lambda}{Nd}$$



- Distribución triangular

$$a_n = \begin{cases} n+1 & n < N/2 \\ n-N & n > N/2 \end{cases}$$

$$|AF(\psi)| = \left| \frac{\sin\left(\frac{(N+1)\psi}{4}\right)}{\sin\left(\frac{\psi}{2}\right)} \right|$$

$$P(z) = \left(\sum_{n=0}^{N-1} z^n \right)^2 = \frac{z^{\frac{N+1}{2}} - 1}{z - 1}$$

$$SLR = \left(\frac{N+1}{2} \right)^2 \left| \sin\left(\frac{3\pi}{N+1}\right) \right|^2$$

$$\lim_{N \rightarrow \infty} SLR = 26'4 \text{ dB}$$

- Distribución binomial

$$a_n = \binom{N-1}{n} = \frac{(N-1)!}{n!(N-1-n)!}$$

$$|AF(\psi)| = \left| 2 \cos\left(\frac{\psi}{2}\right) \right|^{N-1}$$

$$P(z) = (z+1)^{N-1}$$

$$SLR = \infty \quad (\text{No hay lob. secund.})$$

- Directividad del array

$$D = \frac{1}{2kd} \int_{\alpha-kd}^{\alpha+kd} \frac{|AF(\psi)|^2}{AF_{max}^2} d\psi$$

Si $d = n \frac{\lambda}{2}$ ó $kd \rightarrow \infty$ } $\Rightarrow D = \frac{\left(\sum_{n=0}^{N-1} a_n \right)^2}{\sum_{n=0}^{N-1} a_n^2}$

• Máxima directividad

Distribución uniforme endfire $\Rightarrow \alpha_{opt} = -kd - \frac{\pi}{N}$

$$\Delta_{-3dB} \approx \sqrt{1.11} \frac{\lambda}{Nd} \quad D \approx 7.28 \frac{Nd}{\lambda}$$

- Síntesis de agrupaciones

• Dadas las direcciones de los ceros de radiación

$$P(z) = \prod_{n=1}^{N-1} (z - e^{j\psi_{cn}})$$

Si hay dos ceros complejos conjugados

$$(z - e^{j\psi})(z - e^{-j\psi}) = z^2 - 2z \cos(\psi) + 1$$

• Síntesis por Fourier

= Diagrama de radiación $AF(\theta)$



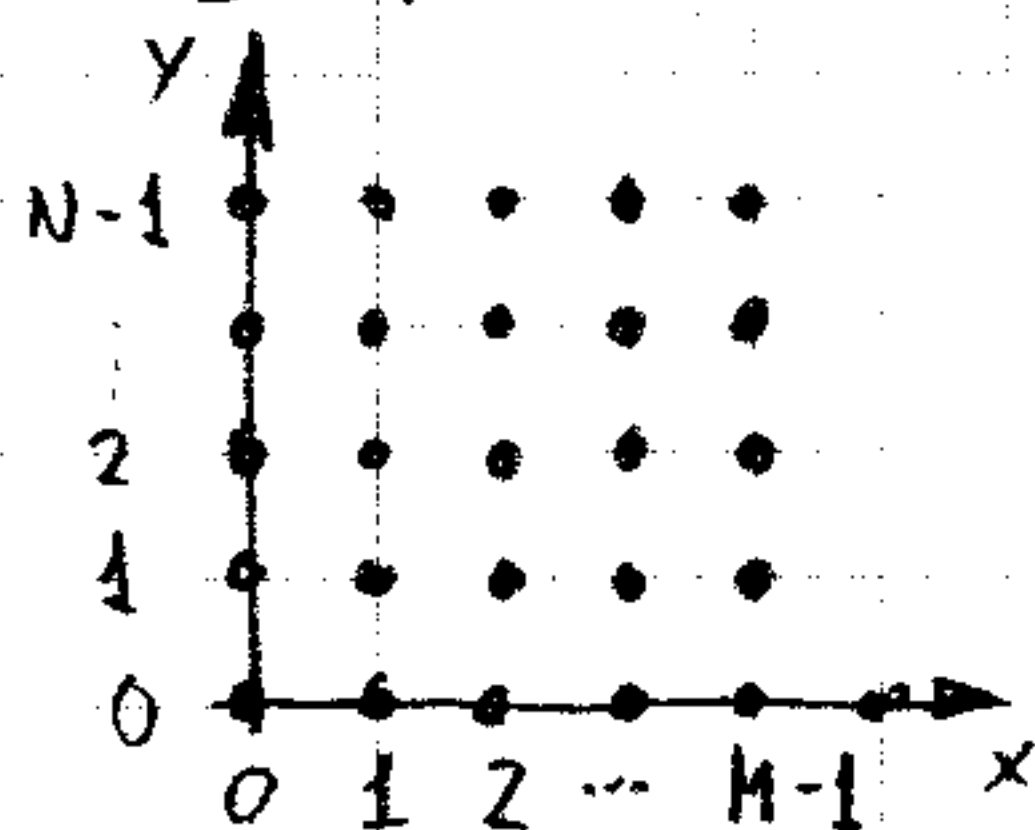
= Factor de agrupación $AF(\psi)$



= Antitransformada de Fourier

$$a_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} AF(\psi) e^{-jn\psi} d\psi$$

- Agrupaciones separables

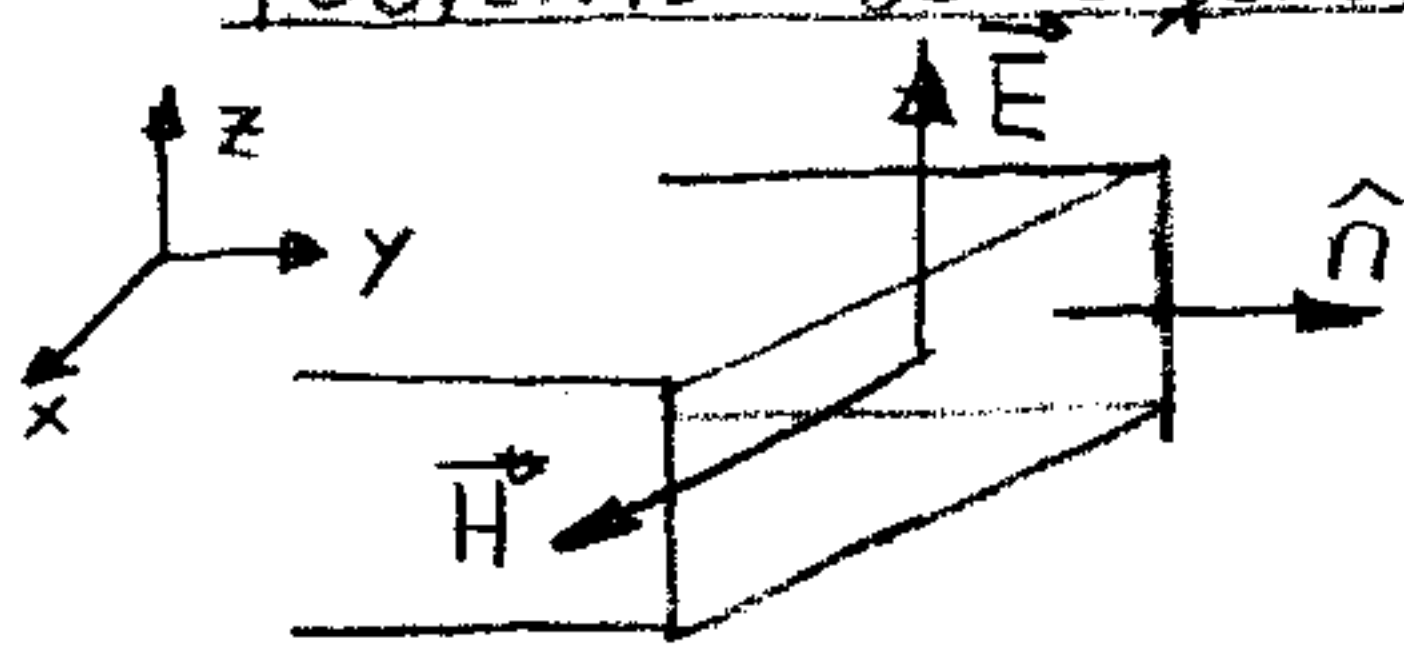


Si las alimentaciones son separables $a_{nm} = a_n a_m$

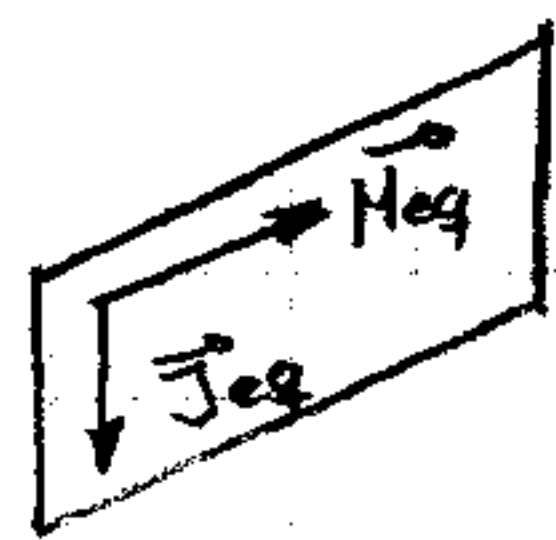
$$AF(\psi_x, \psi_y) = AF(\psi_x) AF(\psi_y)$$

T-4 ANTENAS DE APERTURA

- Teorema de equivalencia



≡



$$\begin{aligned} \vec{J}_{eq} &= \hat{n} \times \vec{H} \\ \vec{M}_{eq} &= -\hat{n} \times \vec{E} \end{aligned}$$

- Expresiones generales de los campos radiados

Para $\vec{E} = E_x \hat{x}$ $\vec{H} = H_y \hat{y} = \frac{E_x}{Z_0} \hat{y}$

$$E_\theta = j \frac{e^{-jkr}}{2\lambda r} \left(1 + \frac{r}{Z_0} \cos\theta \right) \cos\phi \iint_{s_0} E_x e^{jk_x x'} e^{jk_y y'} dx' dy'$$

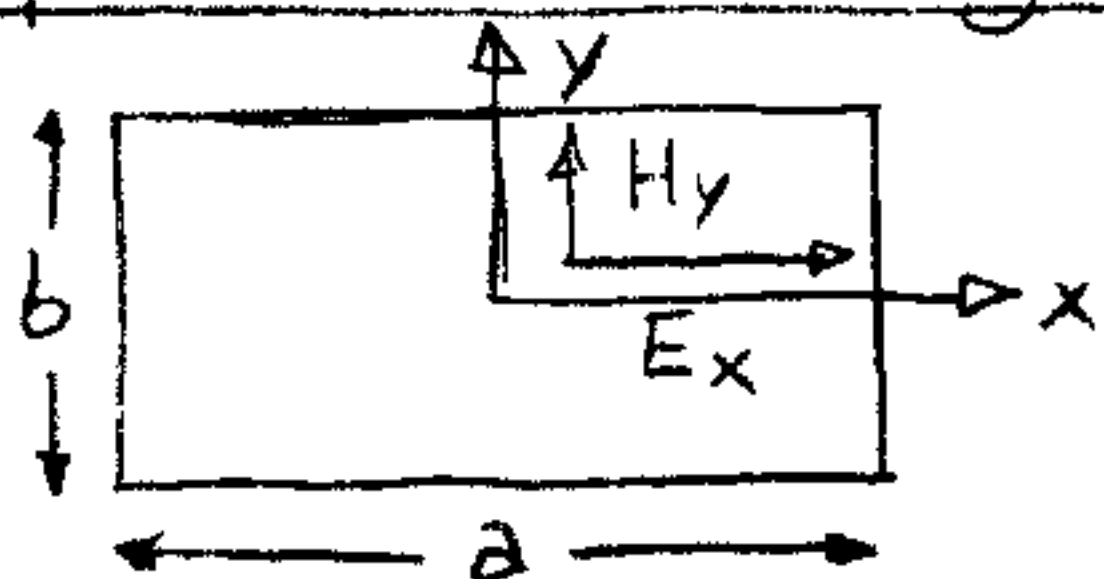
$$E_\phi = -j \frac{e^{-jkr}}{2\lambda r} \left(\frac{r}{Z_0} + \cos\theta \right) \sin\phi \iint_{s_0} E_x e^{jk_x x'} e^{jk_y y'} dx' dy'$$

$$D = \frac{4\pi}{\lambda^2} S \cdot \eta_{il}$$

$$\eta_{il} = \frac{A_{ef}}{\text{Superficie}}$$

$$A_{ef} = \frac{\left| \int_{s_0} E_x ds \right|^2 + \left| \int_{s_0} E_y ds \right|^2}{\int_{s_0} |E_x|^2 + |E_y|^2 dx dy}$$

- Apertura rectangular



$$E_x = E_0 f(x') g(y')$$

$$F(k_x, a) = \int_{-\frac{a}{2}}^{\frac{a}{2}} f(x') e^{jk_x x'} dx'$$

$$E_\theta = j E_0 \frac{e^{-jkr}}{2\lambda r} \left(1 + \frac{r}{Z_0} \cos\theta \right) \cos\phi F(k_x, a) G(k_y, b)$$

$$E_\phi = -j E_0 \frac{e^{-jkr}}{2\lambda r} \left(\frac{r}{Z_0} + \cos\theta \right) \sin\phi F(k_x, a) G(k_y, b)$$

$$\eta_{il} = \eta_x \cdot \eta_y$$

$$\eta_x = \frac{\left| \int_{-1/2}^{1/2} f(s) ds \right|^2}{\int_{-1/2}^{1/2} |f(s)|^2 ds}$$

$$s = \frac{x'}{a}$$

- Apertura uniformemente iluminada

$$E_x = E_0 \cdot 1 \cdot 1$$

$$Z_0 \approx \eta$$

$$D = \frac{4\pi}{\lambda^2} a \cdot b$$

$$\Delta\theta_{3dB}^E \approx 50 \lambda/b \text{ [}^\circ\text{]}$$

$$\Delta\theta_{3dB}^H \approx 50 \lambda/a \text{ [}^\circ\text{]}$$

$$NLPS^E = NLPS^H \approx 13.2 \text{ dB}$$

- Apertura iluminada con el modo TE₁₀

$$E_y = E_0 \cos \frac{\pi x}{a} \cdot 1 \quad Z_0 = \frac{\eta}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}}$$

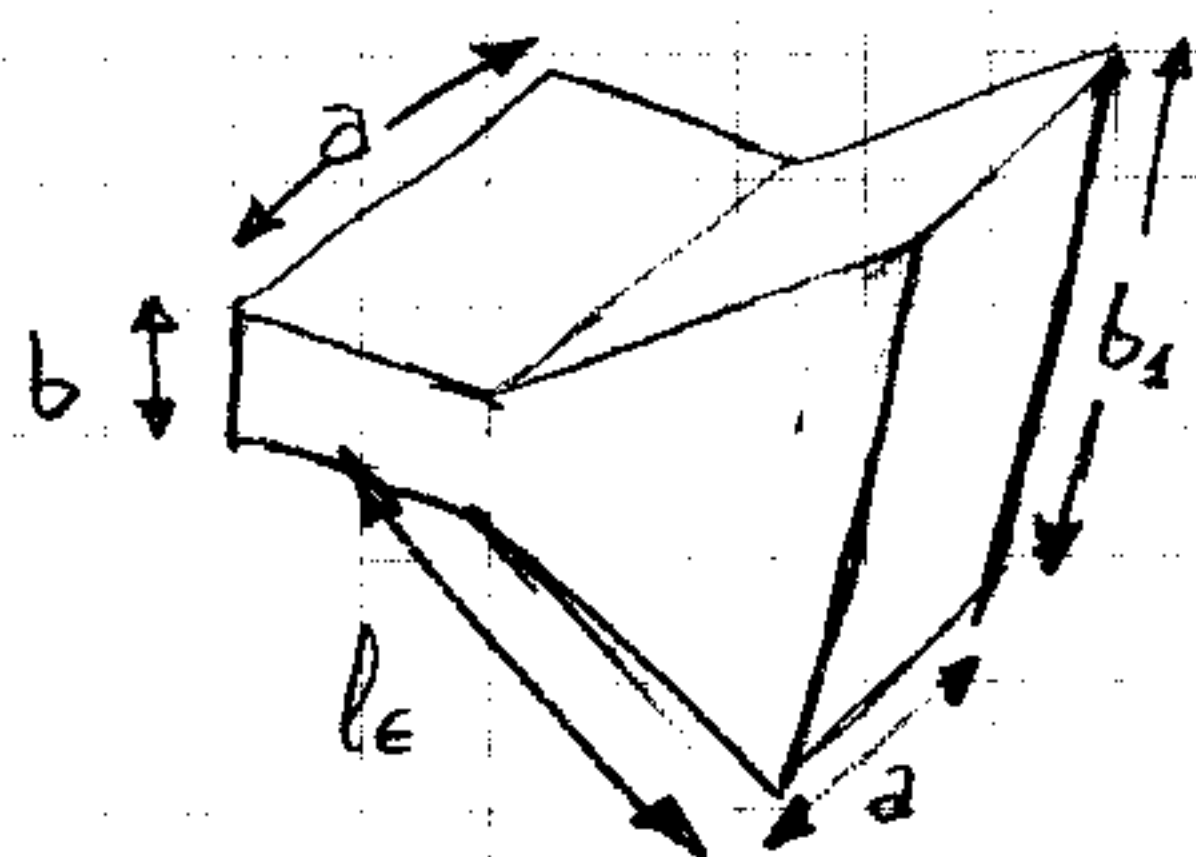
$$\Delta \theta_{3dB}^E \approx 50 \lambda / b \text{ [}^\circ\text{]} \quad \Delta \theta_{3dB}^H \approx 67 \lambda / a \text{ [}^\circ\text{]}$$

$$NLPS^E \approx 13.2 \text{ dB} \quad NLPS^H \approx 23 \text{ dB}$$

$$\eta_{il} \approx 0.81 \Rightarrow D = \frac{4\pi}{\lambda^2} ab 0.81$$

* BOCINAS

- Bocina sectorial de plano E



$$E_y = E_0 \cos\left(\frac{\pi x}{a}\right) e^{-j \frac{\beta y^2}{2le}} \cdot 1$$

$$\beta = k \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}$$

Se usan las fórmulas para aperturas generales

$$P_r = \frac{E_0^2 ab_1}{2\eta}$$

Bocina óptima $\Rightarrow b_1 = \sqrt{2\lambda le}$

$$\Delta \theta_{-3dB}^E \approx 56 \lambda / b_1 \text{ [}^\circ\text{]}$$

$$\Delta \theta_{-3dB}^H \approx 67 \lambda / a \text{ [}^\circ\text{]}$$

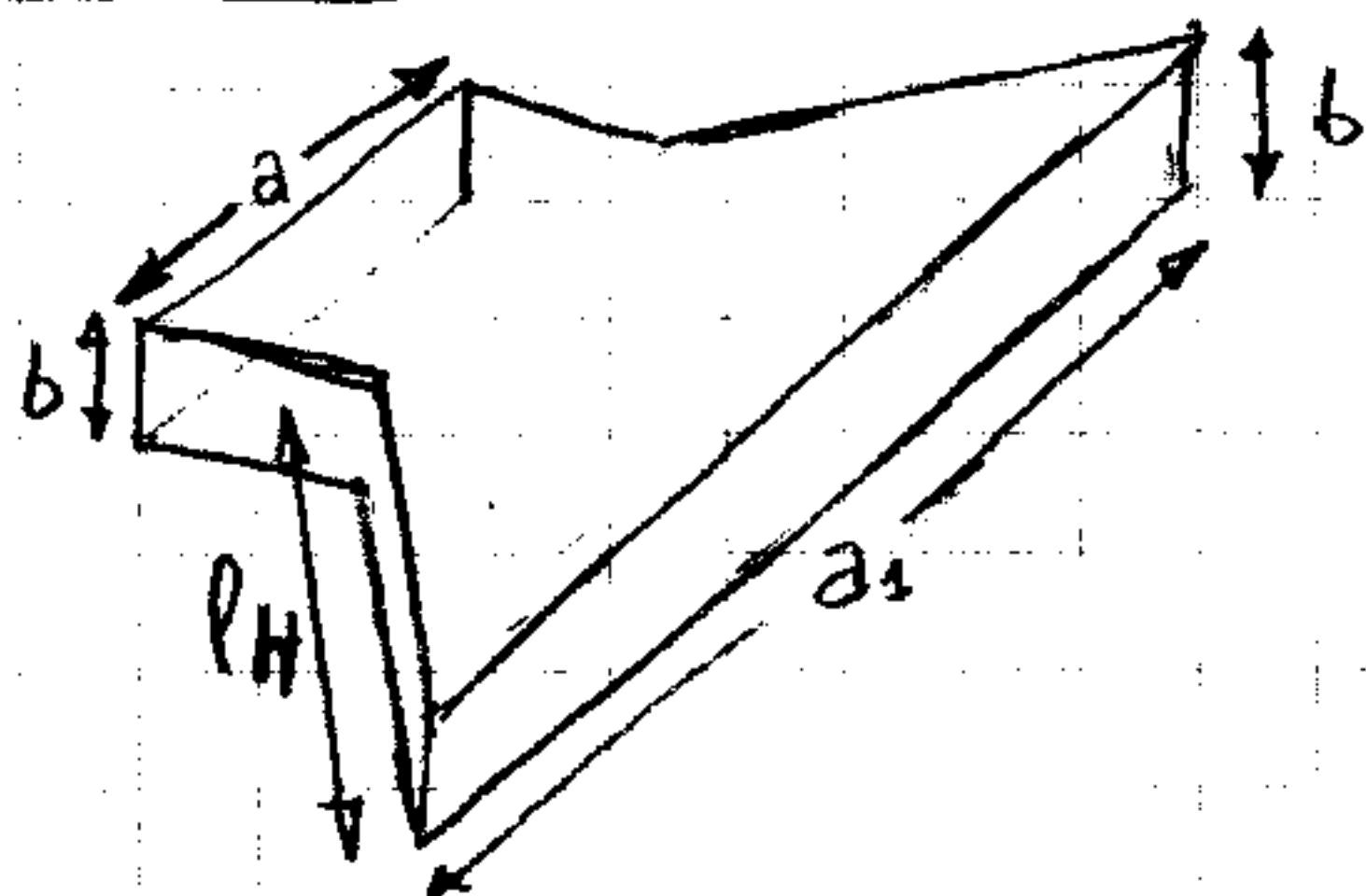
$$NLPS^E \approx 10 \text{ dB}$$

$$NLPS^H \approx 23 \text{ dB}$$

$$\eta_{il} \approx 0.64$$

$$D = 8 \frac{ab_1}{\lambda}$$

- Bocina sectorial de plano H



$$E_y = E_0 \cos \frac{\pi x}{a_1} e^{-j \frac{\beta x^2}{2leH}} \cdot 1$$

Bocina óptima $\Rightarrow a_1 = \sqrt{3\lambda leH}$

$$\Delta \theta_{-3dB}^E \approx 50 \lambda / b \text{ [}^\circ\text{]}$$

$$\Delta \theta_{-3dB}^H \approx 78 \lambda / a_1 \text{ [}^\circ\text{]}$$

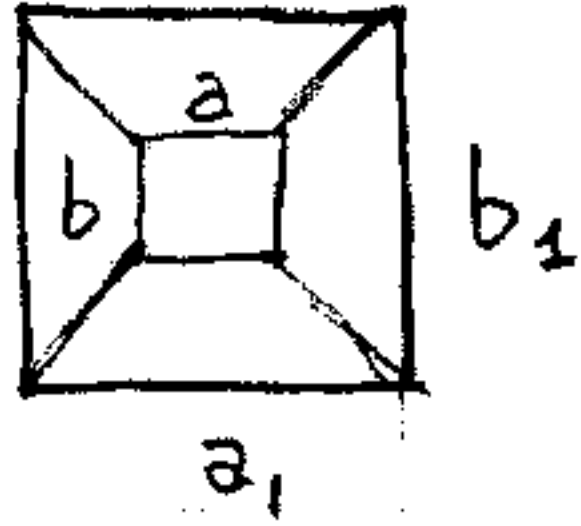
$$NLPS^E \approx 13 \text{ dB}$$

$$NLPS^H \approx 12 \text{ dB}$$

$$\eta_{il} \approx 0.62$$

$$D = 7.9 \frac{a_1 b}{\lambda^2}$$

- Bocina piramidal



$$E_y = E_0 \cos \frac{\pi x}{a_1} e^{j\beta[\delta(x) + \delta(y)]}$$

$$\delta(x) + \delta(y) = \frac{b_1^2}{8l_e} + \frac{a_1^2}{8l_H} - \frac{y^2}{2l_e} - \frac{x^2}{2l_H}$$

$$D_{\text{piramidal}} = \left(D_E \frac{\lambda}{a_1} \right) \left(D_H \frac{\lambda}{b_1} \right) \frac{\pi}{32}$$

Diseño óptimo $\Rightarrow b_1 = \sqrt{2l_e \lambda} \quad a_1 = \sqrt{3l_H \lambda}$

$$\Delta\theta_{-3\text{dB}}^E \approx 56 \lambda / b_1 [^\circ] \quad \Delta\theta_{-3\text{dB}}^H \approx 78 \lambda / a_1 [^\circ]$$

$$\text{NLPS}^E \approx 10 \text{ dB}$$

$$\text{NLPS}^H \approx 12 \text{ dB}$$

$$\eta_{\text{il}} \approx 0.51$$

$$D = 6.4 \frac{a_1 b_1}{\lambda^2}$$

* RANURAS

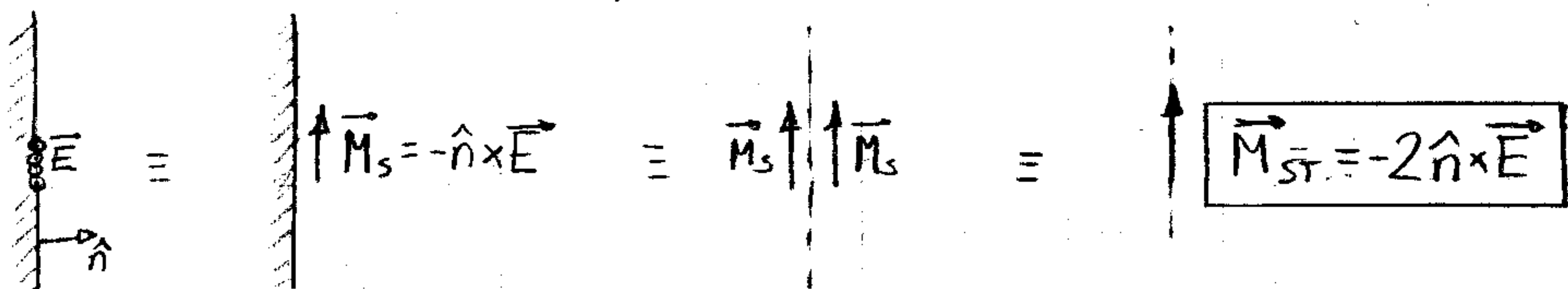
Para cualquier par de antenas complementarias:

$$\boxed{\frac{R_{r, \text{dip}}}{G_{r, \text{ran}}} = \frac{\eta^2}{4}}$$

$$Z_{r, (\text{ran } \lambda/2)} = \frac{\eta^2}{4 Z_{r, (\text{dip } \lambda/2)}} = \frac{\eta^2}{4(73 + j425)} \approx 363 - j211 \Omega$$

$$Z_{r, (\text{doblada})} = \frac{\eta^2}{4 Z_{r, (\text{dip doblado})}} = \frac{\eta^2}{4 \cdot 4 Z_{r, (\text{dip } \lambda/2)}} = 905 - j528 \Omega$$

Las ranuras se analizan a partir de la corriente magnética equiv.



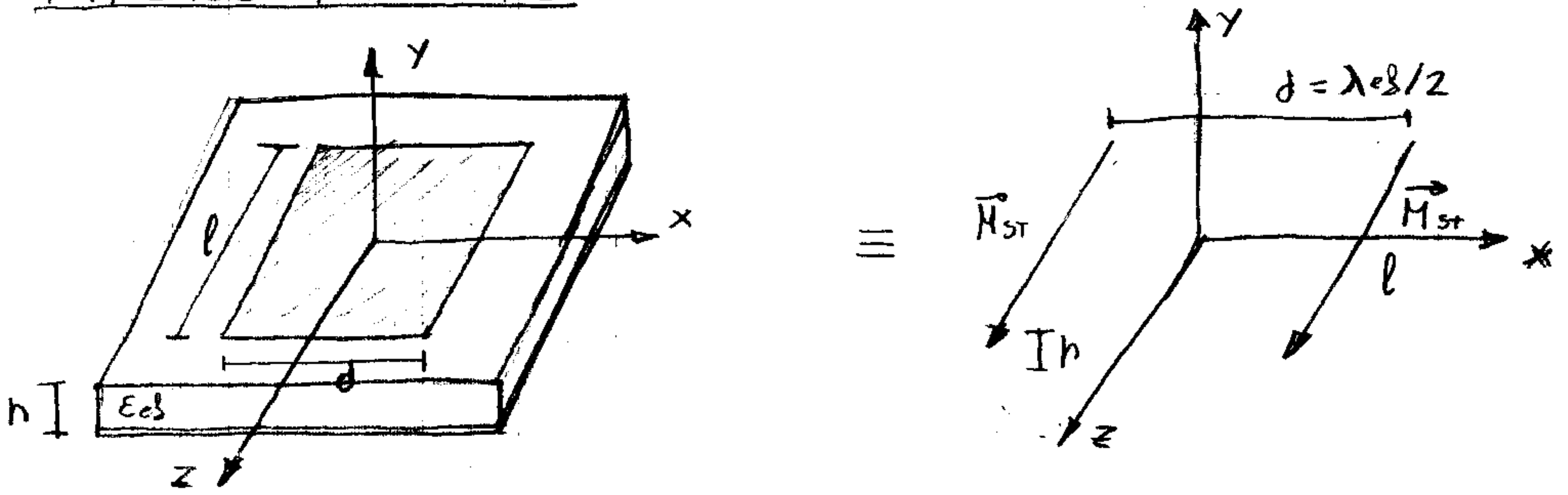
Potencial vector magnético $\vec{F} = \frac{\epsilon}{4\pi r} \int_V \vec{M}_{\text{ST}}(r') e^{jk\hat{r}r'} dv'$

$$\vec{F} = \frac{\epsilon}{4\pi} \frac{e^{-jk r}}{r} \vec{L}$$

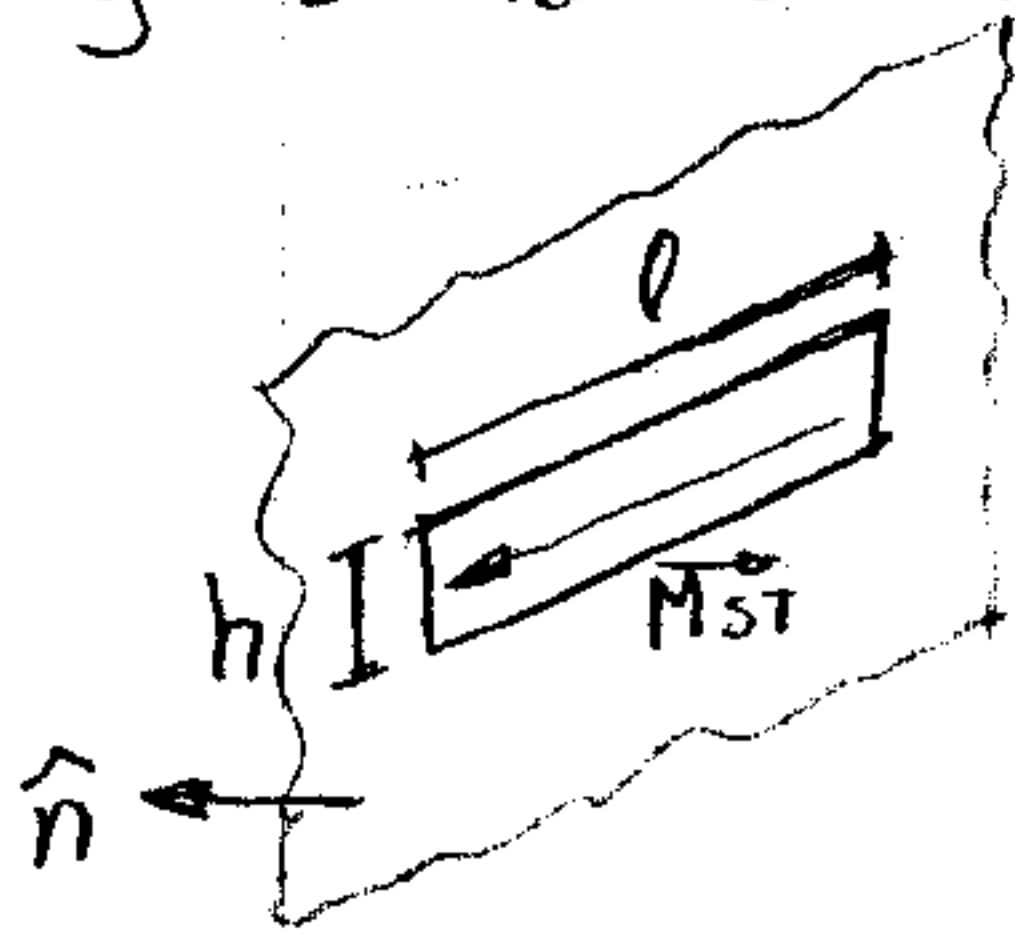
$$\vec{H} = j\omega \hat{r} \times (\hat{r} \times \vec{F})$$

$$\vec{E} = \eta (\vec{H} \times \hat{r})$$

- Antenas microtira



Se puede analizar suponiendo dos ranuras laterales y sumando los campos producidos por cada una



| | | | | |
|--------------------|--------------------|--------------|-----------------------|------------|
| $\eta_x \approx 1$ | ef. polarización | ≈ 51 | $\frac{D_0}{\lambda}$ | \uparrow |
| η_s | ef. desbordamiento | \uparrow | $\frac{D_0}{\lambda}$ | \uparrow |
| η_{il} | ef. iluminación | \downarrow | | \uparrow |

$$D = \frac{4\pi}{\lambda^2} A_{geom} \eta_{il} \eta_x \eta_s$$

* REFLECTORES PARABÓLICOS

- Proceso de diseño: Datos NLPS, η_x , D

